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Mathematical Tables *and other* Aids to Computation

A Quarterly Journal edited on behalf of the
Committee on Mathematical Tables
and Other Aids to Computation
by

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DERRICK HENRY LEHMER

WITH THE COÖPERATION OF

EDWARD WHITNEY CANNON
LESLIE JOHN COMRIE
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III • Number 25 • January, 1949

Published by

THE NATIONAL RESEARCH COUNCIL

NATIONAL RESEARCH COUNCIL
DIVISION OF PHYSICAL SCIENCES

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Published quarterly in January, April, July and October by the National Research Council, Prince and Lemon Sts., Lancaster, Pa., and Washington, D. C.

All contributions intended for publication in *Mathematical Tables and Other Aids to Computation*, and all Books for review, should be addressed to Professor R. C. ARCHIBALD, Brown University, Providence 12, R. I.

Entered as second-class matter July 29, 1943, at the post office at Lancaster, Pennsylvania, under the Act of August 24, 1912.

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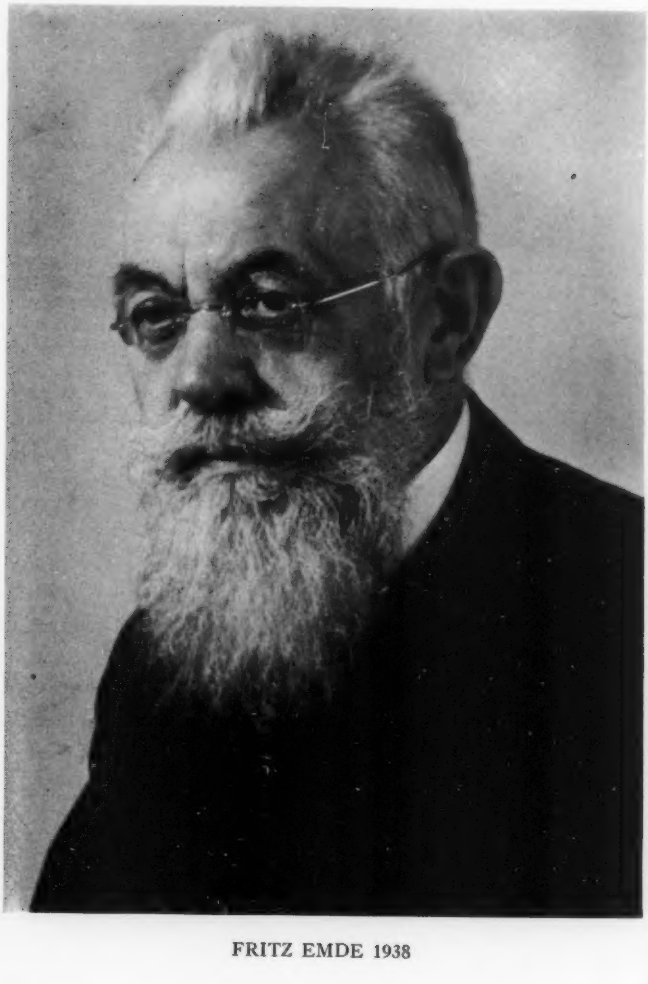


FRITZ EMDE 1938

The BAASMTC now RSMTC

EDITORIAL NOTES:—Dr. MILLER has kindly furnished us with a copy of the final report of the BAASMTC, which has now been replaced by a Mathematical Tables Committee of the Royal Society. He has given us permission to print it almost simultaneously with its appearance in *The Advancement of Science*, the quarterly journal of the BAAS. The long and very interesting report of the late BAASMTC, covering the nine years 1939–1947, was published in this journal, v. 5, Apr. 1948, p. 67–72. From this report we make the following notes:

It is expected that early in 1949 the Committee will publish *Bessel Functions*, Part 2, and at the same time a second edition of Part 1. Part 2, edited by Prof. BICKLEY, will contain 8D or S tables of $J_n(x)$, $Y_n(x)$ or $x^nY_n(x)$, $e^{-x}I_n(x)$ or $x^{-n}I_n(x)$, $e^xK_n(x)$ or $x^nK_n(x)$, for $n = 2(1)20$ and x not greater than 25 or 20; also 10S tables of $J_n(x)$, $Y_n(x)$, $I_n(x)$, $K_n(x)$ for $n = 0(1)20$, and x not greater than 25.

It is planned that *Bessel Functions*, Part 3, shall include tables of: $(2/\pi)K_n(x)$ and $I_n(x)$ for $n = \frac{1}{3}, \frac{2}{3}$; an extension of MEISSEL's table $J_n(x)$ for $x = 1(1)24$, and $n = 1(1)N$, where N is such that $J_{N+1}(x)$ is less than half a unit in the 18th decimal, values to 18D; a similar table of $Y_n(x)$ for the same values of x, and for n up to 30 or more. BESSEL function zeros of $J_n(x)$ and $Y_n(x)$ are planned for Part 4.

The Committee has accepted for publication under the CUNNINGHAM bequest (see *MTAC*, v. 3, p. 144) a compilation by Professor NEVILLE of the FAREY series of order 1025; this series has 319765 terms, and is arranged to occupy 420 pages. The series exhibits solutions of the linear Diophantine equation $bx - ay = 1$ for all values of a and b not exceeding 1025, and furnishes the closest rational approximations within the same order to any number, rational or irrational, between 0 and 1. The introduction shows how by a process of local self-packing the series can be used efficiently to solve linear equations with coefficients of any magnitude and to find rational approximations of any order.

* * *

FINAL REPORT of COMMITTEE on Calculation of Mathematical Tables, Summer, 1948. Dr. A. J. THOMPSON, *Chairman;* Dr. J. WISHART, *Vice-Chairman;* Dr. J. C. P. MILLER, *Secretary;* Prof. W. G. BICKLEY, Dr. R. O. CASHEN, Prof. R. A. FISHER, F. R. S., Dr. E. T. GOODWIN, Dr. J. HENDERSON, Dr. J. O. IRWIN, Dr. C. W. JONES, Prof. L. M. MILNE-THOMSON, Prof. E. H. NEVILLE, Mr. D. H. SADLER, Mr. F. SANDON, Mr. W. L. STEVENS, Mr. JOHN TODD, Dr. M. V. WILKES, and Mr. J. R. WOMERSLEY.

During the past two years, negotiations have been in progress for transferring the responsibility for the work undertaken by the Committee from the British Association to the Royal Society. The Royal Society now has set up a Committee on Mathematical Tables, of which Prof. Bickley, Prof. Fisher, Dr. Miller, Prof. Neville, Mr. Sadler, Dr. Thompson, Dr. Wishart and Mr. Womersley are members. This Committee has set up a General Sub-Committee to advise on and carry out the making and publishing of mathematical tables; an invitation to serve on this Sub-Committee has been extended to other members of the B.A. Committee, who are prepared to play an active part in its work.

The assets and liabilities of the B.A. that are associated with the Committee, including the Cunningham Bequest for new tables in the Theory of Numbers, were transferred to the Royal Society on June 30, 1948, and the B.A. Committee held its final meeting on June 23, 1948.

Seven meetings were held in the past year, during which plans were made for continuing the production of tables under the new conditions; the present status of the various projects in hand is given in the list below. Three volumes, I, VI and IX are to be reprinted shortly, while volume X has been passed for press. Volume X, Bessel Functions, Part II, will be the final volume in the series of B.A. Mathematical Tables.

Of the grant of £150 available this year, £100 has been spent, on Bessel Function checking and calculations, and on the tables of $\tan^{-1} m/n$, etc. The work of volunteers—in

particular of Mr. D. F. Ferguson, of Mr. C. E. Gwyther, of Dr. H. Gupta and of the Mathematics Division of the National Physical Laboratory, through Mr. Womersley and Dr. Goodwin—made it unnecessary to spend the full grant this year.

It seems appropriate to include in this final report a short history of the Committee and of its forerunners, with a list of Chairmen and Secretaries. Following this is a list of publications in book form; mathematical tables given in the Committee's reports are listed in *MTAC*, v. 1, p. 69–75, 1943. The report ends with a list of projects at present in hand; the previous report of the Committee, for the period 1939–47, should be consulted for further details of most of these projects.

Short History of the British Association Mathematical Tables Committee

In 1871, at a meeting of the Association in Edinburgh, a Committee was set up, with A. Cayley (served 1871–95) as Chairman (1871–89), J. W. L. Glaisher (1871–1901) as Secretary (1871–89), and H. J. S. Smith (1871–83), G. G. Stokes (1871–84), and Sir W. Thomson, later Lord Kelvin (1871–1901; Chairman, 1897–1901) for its members. The Committee was formed 'for the purpose of reporting on Mathematical Tables, which it may be desirable to compute or reprint.' The first report of the Committee appeared in 1873 where the aims of the Committee were defined more precisely. 'The purposes for which the Committee was appointed were twofold, viz.: (1) to form as complete a catalogue as possible of existing mathematical tables, and (2) to reprint or calculate tables which were necessary for the progress of the mathematical sciences.'

The first task was carried out magnificently in two parts, by Glaisher for general tables in the 1873 report, and by Cayley for tables in the Theory of Numbers in 1875. These reports have only been superseded within the last 10 years and remain bibliographical works of the greatest value.

The second task, the production of new tables, was also commenced immediately and remains in progress. It was at first intended that tables should be published independently of the Annual Reports of the Association, and this was done with Glaisher's table of e^x and e^{-x}, Camb. Phil. Soc., *Trans.*, v. 13, 1883, p. 243–272. The next works prepared and published by the Committee, with James Glaisher added (served 1877–89), were the *Factor Tables for the Fourth, Fifth and Sixth Millions* (published in 1879, 1880, and 1883), which filled the gap between the tables of Burckhardt and of Dase. The publication was financed by grants from the British Association and the Royal Society. The calculations were chiefly the work of James Glaisher, while J. W. L. Glaisher compiled a comparison of the numbers of primes actually counted in various ranges with the numbers given by three different formulae. The Introduction to the *Sixth Million* contains many similar enumerations of great interest. D. N. Lehmer in his *Factor Table for the First Ten Millions*, says of James Glaisher: 'The character of his work should be a matter of the deepest satisfaction to every member of that Association. The sixth million, in particular, is the most remarkable table of the sort ever published. It contains only one slight error in the computation and two insignificant typographical errors!'

In the B. A. Annual Report for 1879 a table of Legendre Polynomials was given, the first of many various functions that appeared in these Reports up to 1929.

Other tables initiated during the period 1871–88 by J. W. L. Glaisher, were a Table of Powers of Integers, Tables in the Theory of Numbers connected with the Divisors of a Number, and Tables of the Elliptic Theta Functions [see *MTAC*, v. 3, p. 92, 245]. All three reached the stage of printer's proofs, and much mystery is attached to the reasons for not completing the publication at this time. Proof copies of the first two tables were found, and publication, with additions, has recently (1940) been completed in Volumes IX and VIII respectively of the *B.A. Mathematical Tables*. No proof copy of the Elliptic Function tables has come to light and it now seems unlikely that Glaisher's tables will ever be published, since they are not in a form that now seems appropriate and have largely been superseded by the *Smithsonian Elliptic Functions Tables*, by G. W. and R. M. Spenceley [see *MTAC*, v. 3, p. 89f].

In 1888 a fresh Committee was set up with Lord Rayleigh as Chairman (1888–97) and A. Lodge (served 1888–1937) as Secretary (1888–96), other new members being A. G. Greenhill (1888–1901, 1910–21), W. M. Hicks (1888–1901), and B. Price (1888–99). The terms of reference were to consider 'The possibility of Calculating Tables of Certain Mathematical Functions, and, if necessary, of taking steps to carry out the calculations, and to publish the results in an accessible form.' This, broadly, has been the task of the Committee ever since, apart from a period of inactivity from 1901 to 1906. At first the attention of the Committee was directed to the tabulation of Bessel functions, and tables of $I_n(x)$ were given in the report of 1889, $I_1(x)$ in 1893 and $I_0(x)$ in 1896. These were the first of a long series of tables of Bessel functions, culminating in *B.A. Mathematical Tables*, volume VI, Bessel Functions, Part I, published in 1937, and volume X, Bessel Functions, Part II, now in the press, which will terminate the series of *B.A. Mathematical Tables*.

Once again, in 1896, with A. J. C. Cunningham (served 1895–1901) as Secretary (1896–1901), and with P. A. MacMahon (1895–1901) added to the Committee's membership, attention was drawn to a table in the Theory of Numbers, and in 1900 the *Binary Canon*, a new *Canon Arithmeticus*, by A. J. C. Cunningham, was published.

Two other Committees, existing at the same time as the major Committee, may also be considered in some respects as forerunners of the Committee now reporting. When the new Committee was set up in 1888, the old one continued for one year and was then reconstituted for the period (1889–93) with A. Cayley as Chairman, A. Lodge as Secretary, and J. J. Sylvester and A. R. Forsyth as members. Its purpose was to carry on 'the Tables connected with the Pellian Equation from the point where the work was left by Degen in 1817'; the resulting tables appeared in the Report for 1893. The other Committee (1894–99) was set up 'To co-operate with Prof. Karl Pearson in the Calculation of Certain Integrals.' It had Rev. R. Harley as Chairman, A. R. Forsyth as Secretary, and J. W. L. Glaisher, A. Lodge and K. Pearson as members. Tables connected with the $G(r, \nu)$ or Pearson Integrals, computed under the supervision of this Committee, appeared in 1896 and 1899.

After a lull in activity, the Committee was again set up in 1906, with M. J. M. Hill as Chairman (1906–19), L. N. G. Filon (1906–29) as Secretary (1906–10), and A. Lodge as the only member, and the sole link, at that time, with the old Committees. Its purpose was 'The further tabulation of Bessel Functions'; this was altered in 1913 to the 'Calculation of Mathematical Tables,' which has remained the purpose of the Committee since then without a break. A report, dealing with the asymptotic series for $J_n(x)$ and including short tables, was given in 1907. In 1909, the Committee, with the addition of J. W. Nicholson (served 1908–31, Secretary 1910–20, Chairman 1920–31) made a second report, and stated that 'they are also considering the advisability of collecting all existing tables of Bessel Functions and publishing them as a single set of tables in a form easily accessible to all students,' and the following year it was reported that a list of Bessel functions tables had been completed. In 1911 the report dealt with the further tabulation of Bessel and other functions; A. G. Greenhill, who had rejoined the Committee, also brought forward a scheme for the rearrangement of tables of elliptic functions. In the same report tables of $G_n(x)$ and $Y_n(x)$ by J. R. Airey (1911–37, Secretary 1920–29) were published. During the next few years tables of Bessel functions of various types, and of elliptic functions occupied the attention of the Committee. By 1915 the Committee had become much larger with the addition of A. G. Webster (1912–25), E. W. Hobson (1913–29), A. E. H. Love (1913–31), H. M. Macdonald (1913–30), T. W. Chaundy (1914–28), A. T. Doodson (1915–17 and 1925–31), and H. G. Savidge (1915–16); several of the new members had made contributions to the tables of Bessel functions. In the report of this year it was stated, 'the order of calculation is being arranged in accordance with the real urgency of the tables, and the stage is now coming in sight at which the Committee will be able, as authorised already by the Association, to publish a volume of fairly complete tables of the more important transcendental functions.'

In 1916 G. Kennedy (1916–25) and G. B. Mathews (1916–22) joined the Committee, and the report contained tables of sines and cosines of angles in radians (J. R. Airey),

logarithmic Gamma function and derivative (G. N. WATSON, served 1916–25), Bessel and NEUMANN functions, etc. (J. R. Airey, A. T. Doodson). Elliptic functions were dealt with in the Report of 1919, and a table, by R. L. HIPPISLEY (1919–23), was given. In 1920 Nicholson became Chairman and Airey Secretary. During the period to 1929 the reports contained tables connected with Bessel functions, LOMMEL-WEBER functions, confluent hypergeometric functions, FRESNEL's integrals, hyperbolic sines and cosines, exponential, sine and cosine integrals. During this period, new members of the Committee were R. A. Fisher (1925–48), J. Henderson (1927–48), Miss D. M. WRINCH (1923–29).

From 1928–31 the Committee was reorganised, E. H. Neville (served 1929–48) becoming Chairman (1931–47), and L. J. COMRIE (served 1928–37) becoming Secretary (1929–37). Other new members were J. O. IRWIN (1928–48), A. J. Thompson (1928–48), J. F. TOCHER (1928–45), T. WHITWELL (1928–31), J. Wishart (1928–48, Secretary, 1937–46), E. S. PEARSON (1930–33) and FRANK ROBBINS (1930–45). In 1928 the decision was made to discontinue the publication of tables in the B.A. Annual Reports, and to give in book form a number of the tables that had appeared in the reports; gaps were to be filled by new calculations, while provision was to be made for interpolation. The first result of this policy was volume I in the series of Mathematical Tables, which appeared in 1931. It was edited by J. Henderson and includes circular and hyperbolic functions, with material from the Reports of 1916, 1923, 1924, and 1928 by Doodson and Airey, collated and completed by Comrie; Exponential, Sine, and Cosine Integrals with tables by Airey (from reports of 1927 and 1928) and Fisher, collated by Henderson; Factorial and Polygamma Functions, with tables by Watson (from the 1916 Report), Lodge (1929 Report) and Fisher, collated and completed by Lodge and Wishart; *Hh* functions, or integrals and derivatives of the probability integral, with tables by Airey from the 1928 Report, slightly extended, collated by Irwin, with an account of properties and applications by Fisher. This initiated the period of greatest activity in the Committee's history. The Committee has always owed a great deal to its secretaries. The influence of Glaisher and of Airey, in particular, is well exhibited in the progress of the Committee during their periods of office, while Comrie, by the successful application of commercial machines to the construction of mathematical tables, combined with drive and opportunity to push projects to completion, contributed largely to the rapidity with which the Committee's volumes were produced from 1931 onwards. This together with particular attention on the part of the Committee to the special typographical problems connected with the printing of numerical tables, and with the emphasis on the needs of the user who may be consulting such tables continuously for long periods, has resulted in standards of accuracy and presentation that have not been surpassed and rarely equalled.

Volume II, EMDEN Functions, was prepared by D. H. Sadler (served 1932–48), who edited it, and J. C. P. Miller (1933–48, Secretary 1946–48), at the suggestion of Sir ARTHUR EDDINGTON, and financed jointly by the British Association and the International Astronomical Union. Comrie and Airey were responsible for planning the initial stages of the project, and for devising methods of computation. The resulting volume was published in 1932.

Lt.-Col. A. J. C. Cunningham, who died in 1928, left to Section A of the British Association a legacy for producing new tables in the Theory of Numbers. The duty of making use of this bequest was entrusted to the Committee in 1929, and five volumes coming within its terms have so far been produced. Volume III, published in 1933, gives a table of Minimum Decompositions into Fifth Powers, computed by L. E. DICKSON and accepted for publication by the Committee; this is the only volume not produced by printing from type—it was in fact reproduced by photography from typescript—and the results have discouraged a repetition of the experiment, and have reinforced the firm opinion of the Committee that first-class results can be obtained only by printing from type. Volume IV, published in 1934, gives Cycles of Reduced Ideals in Quadratic Fields; this was suggested by W. E. H. BERWICK and computed for the Committee by E. L. INCE (served 1932–41). Volume V, published in 1935, is a Factor Table, prepared independently in triplicate by J. PETERS, by A. Lodge

and Miss E. J. Ternouth, and by Mrs. E. Gifford; the part played by Comrie in collating the three calculations, and in supervising the reading of proofs and the comparisons with published tables was considerable. All three volumes contain the results of new calculations.

The Cunningham Bequest also financed the publication of Volumes VIII, Number-Divisor Tables, and IX, Table of Powers, both of which originate, as remarked above, in tables computed under the direction of J. W. L. Glaisher on behalf of the first Committee about 70 years ago. With proof copies as basis, the tables were checked and extended—volume VIII, by Prof. D. H. Lehmer (who attended meetings of the Committee by invitation in 1939) and J. Wishart (Editor), and volume IX by W. G. Bickley, C. E. Gwyther, J. C. P. Miller (Editor), and Miss E. J. Ternouth. Both volumes were published in 1940.

The production of these volumes has not exhausted the bequest, and further projects connected with the Theory of Numbers have been described in the Committee's previous report. It is intended that these should be completed within the next few years.

In 1929, the decision was taken to separate consideration of Bessel functions from that of other functions, and Bessel functions were made the special business of a sub-committee, whose reports and recommendations have formed the basis of the Committee's subsequent discussions and decisions. The original members of the Sub-Committee were J. Henderson and J. O. Irwin; later members were J. C. P. Miller, D. H. Sadler, A. J. Thompson and W. G. Bickley (served 1934–48), Henderson being Chairman. L. J. Comrie, while Secretary of the main Committee, kept in closest touch with the Sub-Committee and attended many of its meetings. In 1937, volume VI was published; this is Bessel Functions, Part I, (Functions of Order Zero and Unity), and, although consisting largely of new calculations, is a result of the continued interest of the Committee since 1888 in Bessel functions. The volume was dedicated to Prof. Alfred Lodge, secretary of the 1888 Committee, whose record of continuous service is unequalled; Lodge died just before publication, in ignorance of this token of the Committee's appreciation of his work.

The actual preparation of the tables was supervised by Comrie, who also provided an account of the various operations performed and of the final checking of the tables.

Further work on Bessel functions has been subdivided, and the preparation of Part II, (Functions of Integer Order), to form volume X (and last) of the B.A. Series, has progressed steadily, although much delayed by the war; the calculations have been performed under the supervision of Bickley, Comrie, Miller, Sadler and Thompson. The Bessel Functions Sub-Committee was reappointed this year with W. G. Bickley as Chairman, E. T. Goodwin (who joined the Committee in 1947) as Secretary, and C. W. Jones (who also joined in 1947), J. C. P. Miller, D. H. Sadler and A. J. Thompson as members. It is hoped that this sub-committee will be able to carry forward its plans for further tables of Bessel functions under the Royal Society Committee.

Volume VII, *The Probability Integral*, by W. F. Sheppard, was published in 1939 as a memorial to Sheppard. It originated in a plan by Sheppard that was not quite complete at his death. It was edited by J. O. Irwin.

The work of the Committee during the period 1928–41 was much accelerated by the employment of paid computers, at first by Comrie when Secretary; later the Committee as a whole employed a full-time computer, who worked at the Galton Laboratory, by invitation of R. A. Fisher, under the supervision of W. L. Stevens (served 1936–48) and D. H. Sadler. The three full-time computers were Mr. F. Cleaver, Dr. H. O. Hartley and Mrs. R. St. H. Tysser; Mrs. Tysser, on resigning the position of computer soon after the outbreak of war, joined the Committee (as Dr. R. O. Cashen (1941–48)). It is hoped that this very successful experiment of employment of a full-time computer may be repeated by the Royal Society Mathematical Tables Committee.

The appearance of volumes VIII and IX in 1940 was followed by a war-time lull, during which progress was slow, but not negligible. In 1946, a second edition of volume I appeared, and also two 'Part Volumes': A, giving Legendre Polynomials, computed by L. J. Comrie, and edited by A. J. Thompson, and B, giving Tables of the Airy Integral, by J. C. P. Miller, prepared at the suggestion of H. Jeffreys, the work being initiated by L. J. Comrie.

During the period from 1937 onwards, other new members of the Committee were F. Sandon (1938–48), M. V. Wilkes (1938–48), L. M. Milne-Thomson (1939–48), John Todd (1944–48), J. R. Womersley (1944–48).

During the last two years the Committee has been developing the plans outlined in the previous report (for 1939–47) and adding to the list. This report ends with a list of projects in hand and proposed for future work, with an indication of their present status.

The Committee has worked to produce a series of fundamental tables of high accuracy, taking account of practical needs when these were clear, but largely following the inclinations and enthusiasms of its individual members. Its concern has been as much with the technique of table-making as with the results obtained, and it has felt that it was performing a useful function in developing methods and setting standards. If five of the volumes published since 1931 are connected with the Theory of Numbers, this is because the Cunningham Bequest made them possible; examination of the list of projects in hand will show that the Committee's interests have been wide and that the tables ultimately to be published will combine with earlier ones to form a well-balanced set.

In preparing tables for publication and in seeing them through the press, the main aims of the Committee may be summarised as follows: complete accuracy within stated limits (e.g. within 0.52 of the last digit given), full provision for interpolation wherever feasible and relevant, and the highest standards of typography and arrangement. These have been the occasion of numerous and lengthy discussions in Committee, and the standards attained have not been surpassed. New devices of great power for interpolation have been introduced—in particular, modified differences in volume I, with extended use in Part-Volume B.

Chairmen

A. Cayley, 1871–1889.
Lord Rayleigh, 1888–1897.
[1] A. Cayley, 1889–1893.
[2] Rev. R. Harley, 1894–1899.
Lord Kelvin, 1897–1901.
M. J. M. Hill, 1906–1919.
J. W. Nicholson, 1920–1931.
E. H. Neville, 1931–1947.
A. J. Thompson, 1947–1948.

Secretaries

J. W. L. Glaisher, 1871–1889.
A. Lodge, 1888–1896.
[1] A. Lodge, 1889–1893.
[2] A. R. Forsyth, 1894–1899.
A. J. C. Cunningham, 1896–1901.
L. N. G. Filon, 1906–1910.
J. W. Nicholson, 1910–1920.
J. R. Airey, 1920–1929.
L. J. Comrie, 1929–1937.
J. Wishart, 1937–1946.
J. C. P. Miller, 1946–1948.

[1] Pellian Equation Committee.
[2] Pearson Integrals Committee.

B. A. Mathematical Tables Committee

Separate Publications

Factor Table for the Fourth Million. By James Glaisher. 1879, 52, [112] p.
Factor Table for the Fifth Million. By James Glaisher. 1880, 12, [112] p.
Factor Table for the Sixth Million. By James Glaisher. 1883, 106, [112] p.
A Binary Canon. By A. J. C. Cunningham. 1900, viii, 172 p.

B. A. Mathematical Tables

V. I. *Circular and Hyperbolic Functions, Exponential, Sine and Cosine Integrals, Factorial (Gamma) and Derived Functions, Integrals of Probability Integral*. London, B. A. Office, 1931, xxvi, 72 p.

V. I. *Circular and Hyperbolic Functions, Exponential, Sine and Cosine Integrals, Factorial Function and Allied Functions, Hermitian Probability Functions*. Second ed. Cambridge University Press, 1946, xii, 72 p.

V. II. ***Emden Functions.*** By D. H. Sadler and J. C. P. Miller. London, B. A. Office, 1932, viii, 34 p.

V. III. ***Minimum Decompositions into Fifth Powers.*** By L. E. Dickson. London, B. A. Office, 1933, vi, 370 p.

V. IV. ***Cycles of Reduced Ideals in Quadratic Fields.*** By E. L. Ince. London, B. A. Office, 1934, xvi, 80 p.

V. V. ***Factor Table, giving the Complete Decomposition of all numbers less than 100,000.*** By J. Peters, A. Lodge and E. J. Ternouth, E. Gifford; collated and edited by L. J. Comrie. London, B. A. Office, 1935, xvi, 292 p.

V. VI. ***Bessel Functions, Pt. I, Functions of Orders Zero and Unity.*** Edited by J. Henderson, tables compiled and described by L. J. Comrie. Cambridge University Press, 1937, xx, 288 p.

V. VII. ***The Probability Integral.*** By W. F. Sheppard. Edited by J. O. Irwin. Cambridge University Press, 1939, xii, 34 p.

V. VIII. ***Number-Divisor Tables.*** By J. W. L. Glaisher. Completed by D. H. Lehmer and J. Wishart (Editor). Cambridge University Press, 1940, x, 100 p.

V. IX. ***Table of Powers,*** giving Integral Powers of Integers. By J. W. L. Glaisher, W. G. Bickley, C. E. Gwyther, J. C. P. Miller (Editor), E. J. Ternouth. Cambridge University Press, 1940, xii, 132 p.

Pt.-V. A. ***Legendre Polynomials.*** By L. J. Comrie. Edited by A. J. Thompson. Cambridge University Press, 1946, 42 p.

Pt.-V. B. ***The Airy Integral.*** By J. C. P. Miller. Cambridge University Press, 1946, 56 p.

Auxiliary Tables (on card), prepared by J. C. P. Miller.

Number 1. Coefficients in the Modified Everett Interpolation Formula, 1946.

Number 2. Table for Interpolation with Reduced Derivatives, 1946.

In the Press

V. X. Bessel Functions, Part II, Functions of Positive Integer Order 2 to 20. By W. G. Bickley (Editor), L. J. Comrie, J. C. P. Miller, D. H. Sadler, A. J. Thompson. About 300 p.

V. I, VI, IX. Further editions or reprints are in process of preparation.

Projects in Hand, for Future Consideration by the Royal Society Mathematical Tables Committee

Most of these projects have already been described fairly fully in the 1939–47 report of the Committee, and will only be noted briefly below. Numbers 1, 3, 5 and 8 are to be published by means of the Cunningham Bequest and are expected to exhaust it.

Tables completely or almost completely planned, and for which considerable work has been done

1. *The Farey Series,* F_{1025}. Compiled and edited by Prof. E. H. Neville. See previous report. Manuscript and preliminaries complete. About 420 p.
2. *Bessel Functions of Orders* $\pm\frac{1}{4}$ and $\pm\frac{3}{4}$. Initiated by Dr. L. J. Comrie, edited by Mr. D. H. Sadler. See previous report. Computations almost completed.
3. *Binomial Coefficients*. The origin of this project is the Table of Binomial Coefficients offered by Mr. W. E. Mansell and mentioned in the last report. A pagination scheme has been drawn up, and the preparation of new material and of printer's copy is in progress.
4. *Fundamental Tables of Bessel Functions to many decimal places*. This includes the extension of Meissel's table of $J_n(x)$ mentioned in the previous report. Computations by Mr. C. E. Gwyther and Dr. J. C. P. Miller have reached an advanced stage. The N. P. L. Mathematics Division is assisting with final stages in the calculations.

5. *Partition Tables*. By Prof. H. Gupta, with extensions by Mr. C. E. Gwyther and Dr. J. C. P. Miller. See previous report. Prof. Gupta has extended his table for $m \leq 50$ to $n = 400$, and has offered to prepare copy for the printer. A pagination scheme has been prepared.
6. *Cartesian to Polar Conversion Tables*. Supervised by Prof. E. H. Neville. To give, for integral values of x, y, with $y \leq x \leq 105$, values to 12 figures of r with θ in degrees and of $\ln r$ with θ in radians. Values of r not hitherto available are being computed at the Mathematics Division of the N. P. L., and values of θ by Mr. S. Johnston and others; the table incorporates unpublished results by Miss E. J. Ternouth and Mr. S. Johnston.

Tables agreed by the Committee, at least in principle, and for which considerable work has been done

7. *Bessel Function Zeros*. Supervised by Prof. W. G. Bickley, Dr. C. W. Jones and Dr. J. C. P. Miller. See previous report.
8. *Coefficients in Powers of Euler's product*. $F(q) = \prod_{k=1}^{\infty} (1 - q^k)$. See previous report. Computations in progress by Mr. D. F. Ferguson and Dr. J. C. P. Miller. It is also proposed to give coefficients in powers of $F(q)F(q^2)$, of $F(q)F(q^3)$, etc.

Projects on which some work has been done, but which have not reached the stage of formal adoption

9. *Bessel Functions of half-integer order*. Values calculated for I_n and K_n under the supervision of Dr. J. C. P. Miller are offered by Scientific Computing Service. Extension of these tables is in hand under the supervision of Dr. C. W. Jones.
10. *Fundamental Tables for Computers to many figures*. Supervised by Dr. A. J. Thompson. It is proposed to produce a collection giving short tables of functions, mainly elementary, and of constants to a high degree of accuracy. These are intended to meet the occasional needs of computers of mathematical tables who want certain fundamental values to many figures. Members of the Committee have already made contributions; readers who possess suitable material that they are willing to make available are invited to offer them to the Committee.

    Considerable progress has been made with tables of square roots (50 decimals), cube roots (20 decimals), fourth and twelfth roots, by Mr. D. F. Ferguson, Mr. C. E. Gwyther, Mr. S. Johnston and Dr. J. C. P. Miller; with a table of $\log_{10} N$ to 45 decimals and with a table of $\log \Gamma(x)$ to 32 decimals by Dr. A. J. Thompson. A table of $n!$ to $n = 1000$ with 20 figures has also been offered by Mr. S. Johnston.
11. *Confluent Hypergeometric Functions*. Computed by Dr. A. J. Thompson. See previous report.
12. *Struve Functions*. Tables have been prepared by Mr. C. Robinson at King's College, London, for an M.Sc. Thesis, under the general supervision of Dr. J. C. P. Miller. These need extension and subtabulation.

Possible Future Projects

13. *Kelvin Functions*. A programme of computation is under consideration by Mr. G. A. Garreau, Mr. M. Bridger and Mr. G. K. Vincent of Northampton Polytechnic, St. John St., London, E. C. 1.
14. *Integrals of Bessel Functions*. The computation of a number of these is under consideration by Dr. E. T. Goodwin.
15. *Inverse Tangents*. A large number of inverse tangents in radians, have been computed, some in the preparation of the Cartesian to Polar Conversion Table (no. 6) and some for other purposes. The publication of a systematic collection of these tables, as a supplement to no. 6 or otherwise, is under consideration.

Coding of a Laplace Boundary Value Problem for the UNIVAC

General Explanation of the UNIVAC System.—The UNIVAC [1] (Universal Automatic Computer) system includes a high-speed electronic digital computer and certain auxiliary devices. This system, which deals with numerical data in decimal form, and which also can handle alphabetic characters, has been designed as a general-purpose tool for scientific and commercial use. A salient characteristic of the system is its flexibility. In its design, particular attention was given to the needs of the Census Bureau, where sorting and collating of information play a predominant role; and a thorough investigation has demonstrated the suitability of the system for such applications.[2] However, the UNIVAC system is not limited to statistical applications and will be useful in performing complicated numerical computations underlying a vast body of scientific research. It is the purpose of this paper to give an example of the application of this system to the solution of the type of problem just mentioned.

All information to be used by the UNIVAC, which is the central computing unit of the system, is first recorded on magnetic tape. Such tape is prepared on a Unityper, which resembles a standard typewriter. One or more tapes are prepared; one usually contains instructions—another may carry numerical data peculiar to the functions needed in the course of the work. All tapes to be used in a problem are then put on input-output readers controlled by the UNIVAC. Some of the tapes are used to record output data; others may be used for temporary storage of data needed at various intermediate steps of the calculations. Recording of new data on a tape automatically erases any previous record in the interval of the tape being used.

When a problem is completed, either the final results are printed on a directly-connected typewriter or they are put on one or more of the tapes. The directly-connected typewriter is usually used for results comprising a very small amount of data. If the results are on tapes, they are inserted into a Uniprinter. The Uniprinter prints the results while another problem is being solved on the UNIVAC. Proper instructions are inserted in the output tapes to control the Uniprinter for tabs, decimal points, spaces, printing of alphabetic headings, etc. With this feature, the results can be arranged in almost any desired form.

The internal memory of the UNIVAC contains 1000 memory locations numbered from 0000 to 0999. Each memory location accommodates 12D digits. A memory location is usually filled by a signed 11-digit number or two instructions. An instruction is usually a letter (which occupies two digit spaces) followed by four digits, designating the memory location, e.g., B0101. A 12-digit group is referred to as a "word." When information is read from a memory location "m," this information still remains in "m" until new information is sent for replacing the old.

The tape record is partitioned into fixed lengths called "blocks," each of which contains 60 words. The first block is read into the first 60 memory locations in .072 second when a start button is pressed. Then the computer

automatically goes to memory location 0000 for its first instruction. After the first instruction is executed, the next instruction comes from the succeeding memory location, unless a transfer instruction is given. Such a transfer interrupts this sequence and causes the next instruction to be drawn from a new memory location designated by the transfer command. Thereafter instructions are taken in sequence from successive locations until a new transfer is encountered.

All arithmetic operations are carried out in the Accumulator, A, of the UNIVAC and the associated registers and circuits. It should be noted that when a word is read into one of these associated registers, the previous contents of that register are erased. An abbreviated list from the UNIVAC instruction code,[1] including the description of only those instructions which are to be used in the solution of the Laplace boundary value problem treated later in this paper is appended below. In this list of instructions, m is a number designating a memory location; letters preceding m are operation symbols. In the description of each instruction, letters denote registers of the UNIVAC. The contents of a register are indicated by parentheses. Thus (m) signifies "word stored in m," (L) signifies "word stored in L," etc. The key mnemonic words are italicized in the description of the operation.

In denoting the different registers, A signifies accumulator. When cleared, this register contains all decimal zeros, even in the sign position. It has capacity for an 11-digit number and sign; a negative number is registered in it in the absolute-value form with a negative sign. The X register is a 60-pulse delay line with an extra 5-pulse delay which can be switched in for shifting purposes. It receives numbers which are to be sent to the accumulator and provides for the checking of the sign to effect algebraic addition of the number to the contents of the accumulator. The L register is a one-word register which contains the multiplicand during the multiplication process. It also holds the word for the T and Q comparison processes. The I register holds one block of words which has been read in from magnetic tape.

The above-mentioned list of instructions to be used in the subsequent discussion is as follows:

Instruction	Explanation of Instruction
Am	*Add* (m) to (A), result in A; (m) also left in X.
Bm	Clear (A), then put (m) in A; (m) also left in X.
Cm	Put (A) in m, *clear* A.
Hm	Put (A) in m without clearing A (i.e., *hold* (A) in A).
Km	Put (A) in L, clear A, disregard m.
Lm	Put (m) in L; (m) also left in X.
Mm	*Multiply* (m) and (L), rounding off the product to 11 digits and adding it to (A), result in A.
Qm	Transfer control to m if (A) = (L).
Sm	*Subtract* (m) from (A), result in A; −(m) also left in X.
Tm	Test to see if (A) is greater than (L); if so, transfer control to m.
Um	*Unconditional* transfer of control to m.
Xm	Add (X) to (A), result in A, disregard m; (X) unaltered.
00m	Pass on to next order without doing any arithmetic operation; disregard m.

Instruction	Explanation of Instruction
	Shift Orders
Δnm	*Shift* all digits of A, *including the sign*, n digits to the *left*, dropping the n left-hand digits; n ranges from 0 to 9; disregard m.
$.nm$	*Shift* all digits of A, *including the sign*, n digits to the *right*, dropping the n right-hand digits; disregard m.
	Tape Orders for 1 to 9 Tapes
$1nm$	*Read* one block of data (60 words) from tape n and store in I, tape running in *forward* direction; disregard m.
$3nm$	*Transfer data* (60 words) previously stored in I to 60 consecutive memory locations, beginning with m, where m is an integral multiple of 20; then *read* one block of data (60 words) from tape n and store in I, tape running in a *forward* direction.
$5nm$	*Write* 60 consecutive words, starting with m, where m is an integral multiple of 20, on tape n, tape moving in a *forward* direction.
$4.m$	Stop machine operations and produce a signal; disregard m.

Underlying Mathematical Considerations in the Numerical Solution of a Laplace Boundary Value Problem.—For purposes of simplicity, a two-dimensional potential problem will be considered. The iterative method used here for the solution of the plane potential problem is a finite-difference method originally proposed by LIEBMANN.[3] The UNIVAC is well adapted for the solution of all sorts of problems where iterative procedures are effective, and not only the plane potential problem, but many others of a similar nature are easily dealt with. When an automatic procedure has once been set up for a single equation, any number of iterations may be made according to the same routine; and, although the computing time will increase in direct proportion to the number of iterations to a good approximation, no further human effort is required. An important simplification in the instructions for a single iterative cycle can be made because of the fact that for all interior points of the lattice exactly the same sort of operations must be carried out. Consequently, after the coding applicable to one lattice point has been worked out, this coding may be automatically altered within the computer so as to apply to the next lattice point. The necessary alterations are systematic, and consequently the routine which accomplishes such alterations can easily be generalized so that all of the required alterations throughout the iterative cycle are accomplished properly.

The LAPLACE equation,

$$\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 = 0,$$

together with the values of $W(x, y)$ on a closed boundary in the xy plane, determines the function W at all points on the interior of the boundary. Here it will be assumed that the boundary is a simple rectangle with sides parallel to the x and y coordinate axes. The finite-difference solution deals only with values of W at discrete and equally spaced points, which will be called lattice points. These lattice points are, for convenience, serially numbered in a systematic way. Let the serial number or index be called j. Then the rectangular array of points may be arranged in q columns and p

rows. In terms of p and q, the index j for each lattice point can be exhibited in the following array:

1	2	.	.	$q-1$	q
$q+1$	$q+2$	.	.	$2q-1$	$2q$
.	.	.	.	.	.
.	.	.	.	.	.
$(p-1)q+1$	$(p-1)q+2$	.	.	$pq-1$	pq

The lattice points lying on the boundary are included in this array and appear in the first and last rows and the first and last columns. At these points the value of the function W is specified in advance. Actually, no use is made in the computations of the corner points ($j = 1$, q, $(p-1)q+1$, and pq), but these points are included in the array so as not to disturb the systematic character of the enumeration.

The value of the finite-difference solution W at any point will be denoted by $W(j)$. The value of $W(j)$ at an interior point is related to the values at the adjacent points by the following equation:

$$W(j) = \tfrac{1}{4}[W(j-1) + W(j+1) + W(j-q) + W(j+q)].$$

The functional values in the above equation are those which exactly satisfy the difference equation which is to be solved in lieu of the LAPLACE differential equation; they provide an approximation to the solution of the original equation. By writing out the $(p-2)(q-2)$ linear equations derived in this way for all interior points and solving these by any appropriate method, one could, of course, obtain the solution in a direct manner, but with a great deal of labor. In the Liebmann process, successive approximations to the correct solutions are obtained by a suitable modification of the above equation. Let $W(j)_i$ be the ith approximation to $W(j)$. Then the next approximation is obtained from the equation:

$$W(j)_{i+1} = \tfrac{1}{4}[W(j-1)_i + W(j+1)_i + W(j-q)_i + W(j+q)_i].$$

Actually, it is more convenient in forming the next approximation for any given point to utilize the best approximation so far obtained for the neighboring points; therefore the following equation is the one actually used:

$$W(j)_{i+1} = \tfrac{1}{4}[W(j-1)_{i+1} + W(j+1)_i + W(j-q)_{i+1} + W(j+q)_i].$$

It will be noted that in this latter equation use is made of the $(i+1)$th approximation, which is already available for points $(j-1)$ and $(j-q)$. This assumes that the computations are carried out so as to proceed systematically from the lowest value of j to the highest throughout the lattice. In most cases, somewhat quicker convergence will be found when this formula is used than if the preceding formula is used.

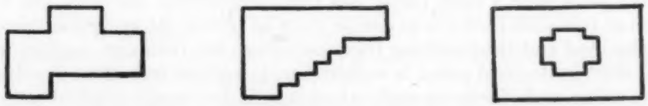
Any desired test of convergence may be used, since the UNIVAC is capable of carrying out any desired comparison process. The test which has been incorporated in the program presented here is merely an example of one such method. At each point and for each iteration, the absolute value

$$|W(j)_{i+1} - W(j)_i| = |\delta|$$

is formed. The sum of these absolute values over all interior points for a single iteration is used in testing convergence. Iteration is continued until this sum is diminished to some satisfactory preassigned value known as the maximum allowable error. (This maximum allowable error should not be taken as zero, since it is possible that because of round-off errors there will be no iterative cycle for which the above sum will reduce to zero; but instead this sum will ultimately vary in the periodic manner with a small amplitude.)

If the higher differences of the function W are not negligible, then the solution to the difference equation will differ appreciably from the solution to LAPLACE'S differential equation. The discrepancy between the two depends, of course, on the fineness of the lattice. Having the solution to the difference equation, one may cause the computer to calculate the appropriate differences in order to estimate whether a finer lattice should be used. Another possible procedure is known as the "deferred approach to the limit." In this case, several solutions of the same problem with different values of p and q are carried out, and from these one may infer the solution for the differential equation by extrapolation. These procedures are not incorporated in the present coding but could easily be handled by the UNIVAC. They are mentioned here only to emphasize the fact that the convergence tests discussed above have to do only with the solution to the difference equation and that avoidance of truncation error is a separate problem.

In the coded routine which follows, a simple rectangular boundary has been assumed. It should be pointed out, however, that modification of this routine to handle somewhat more general boundaries is not difficult. So long as the domain is bounded by vertical or horizontal lines connecting equally spaced lattice points, no particular difficulty is encountered. As examples of such domains the following illustrations are given. (Obviously curvilinear boundaries may be approximated by such rectangular boundaries.)



The same systematic serial numbering of lattice points would be used as in the case of the simple rectangular boundary. In order to make use of the same iteration equation and corresponding coded routine, this numbering should be such that the points which are the neighbors of point j are always $(j - 1)$, $(j + 1)$, $(j - q)$, and $(j + q)$. This is achieved by setting up a simple rectangular lattice which covers the actual domain of interest. Then, just as the corner points of the rectangular lattice are ignored in the simple problem, all other points which lie outside of the domain of interest will also be ignored. This means that in the actual routine for the more complicated cases an additional set of constants must be supplied as part of the initial data of the problem. These constants specify those values of j which mark off the beginning and end of each row in the lattice.

In any numerical work, and particularly in extensive calculations, attention must be given to the magnitudes of the numbers which occur at various points of the work. With the UNIVAC, facilities are provided which make it unnecessary for anyone to estimate what magnitudes are likely to occur or to provide in advance the proper scale factors to prevent numbers from running out of bounds. When numbers exceeding unity may possibly occur, the operator can insert as a part of his problem a subroutine which will automatically introduce appropriate scale factors and carry on the computation correctly. Also subroutines can be used in such a way as to achieve what is known as a "floating decimal point," making it entirely unnecessary for the operator to give any attention to the magnitudes of numbers. In this case all numbers are put in the form $A(10^s)$, where A is a number whose magnitude is less than unity and s is an integer which may range between exceedingly wide limits. The pair of numbers A and s are then used by the computer in conjunction with subroutines which always cause them to be interpreted as $A(10^s)$.

In the plane potential problem presented here, there is no need to resort to a "floating decimal" process nor to make use of the special facilities for accumulator overflow, since a single scale factor applied to the original input data will automatically insure that no number greater than unity will ever occur during the course of the computation. This scale factor is conveniently chosen as an integral power of 10. After applying such a scale factor to all boundary values, no boundary value should have a magnitude greater than .25. For example, if the largest boundary value for a given problem is 625 before scaling, then one should use a scale factor of 10^{-4} so that the boundary value as presented to the computer becomes .0625. Consequently, the sum of four function values which must be formulated during the iterative process will never exceed unity, and since such sums are multiplied by .25 to obtain a new function value, no number exceeding unity will ever occur in the computation.

It is thus apparent that a fixed decimal point is satisfactory for this problem, once such a scale factor has been introduced, and that the introduction of this scale factor is so simple as to be trivial. After convergence has been obtained and the resulting function values are read out, nothing more than a shift in decimal point is required to introduce into these results the compensating scale-factor to make them applicable to the original problem.

Preparation of a Boundary Value Problem for Solution by the UNIVAC System.—In the preparation of a problem for solution by the UNIVAC system, use is made of a typical coding sheet having three columns: the first shows memory locations; the second contains the first half of a word or one instruction; and the third contains the second half of a word or another instruction. It is convenient to stagger the halfwords so as to allow notes to be entered at the right of each instruction.

In the following programming routine, parentheses around an order (e.g., the orders stored in 0001) denote the fact that this order is continually being modified. Also, it should be noted that, at the beginning of the routine, the initial start button is pressed, the first block of orders are read from tape 1 into the I tank and thence to memory cells 0000 to 0060, and the first order stored in cell 0000 is automatically executed.

The actual program for the problem is as follows:

p number of rows
q number of columns
i number of the iteration

			Explanation of Instructions
0000	110000		One block of words from tape 1 to I register.
		320060	Contents of I register transferred to memory cells 60–119 inclusive.
0001	(320100		
		A 0001)	
0002	L 0060		
		Q 0005	Boundary values and initial internal values placed in memory locations 100–999 inclusive.
0003	A 0061		
		C 0001	
0004	000000		
		U 0001	
0005	(B 0101		$W(2)_i$ placed in A.
		A 0130)	$W(q+1)_i$ added to (A).
0006	(A 0132		$W(q+3)_i$ added to (A).
		A 0161)	$W(2q+2)_i$ added to (A).
0007	K 0000		$4W(q+2)_{i+1}$ placed in L; A cleared.
		M 0044	$W(q+2)_{i+1}$ placed in A.
0008	H 0059		(A) stored in 0059; also held in A.
		(S 0131)	$W(q+2)_{i+1} - W(q+2)_i = \delta$ placed in A.
0009	Δ10000		δ shifted one digit to the left; left-hand digit dropped.
		.10000	δ shifted one digit to the right.
0010	A 0058		$\Sigma\lvert\delta\rvert$ placed in A.
		C 0058	$\Sigma\lvert\delta\rvert$ cleared to 0058; A cleared.
0011	A 0059		$W(q+2)_{i+1}$ placed in A.
		(C 0131)	$W(q+2)_{i+1}$ cleared to 0131; replaces $W(q+2)_i$.
0012	A 0005		(B0101 A0130) placed in A.
		A 0045	(B0102 A0131) placed in A; order to be stored in 0005 modified; (0045) in X.
0013	C 0005		(B0102 A0131) cleared to 0005; A cleared.
		X 0000	(0045) now in A.
0014	A 0006		(A0133 A0162) placed in A; order to be stored in 0006 now modified.
		C 0006	(A0133 A0162) placed in 0006; A cleared.
0015	A 0008		
		A 0046	
0016	C 0008		
		X 0000	
0017	A 0011		
		C 0011	
0018	A 0046		
		A 0056	1 added to count of averages.
0019	L 0057		If the count of averages = the count denoting end of row
		Q 0021	$(n(q-2))$, control transferred to 0021.
0020	C 0056		Count cleared to 0056.
		U 0005	Return to instruction sequence for computing averages ($W(j)_i$ values) of next row.

Explanation of Instructions

0021	L 0054	Q 0030	If iteration has been completed (i.e., values of $W(j)_i$ for 784 values of j completed), control transferred to 0030.
0022	A 0053	C 0057	$q - 2$ added to count in 0057.
0023	A 0005	A 0047	3 added to labels at row end.
0024	C 0005	X 0000	
0025	A 0006	C 0006	
0026	A 0008	A 0048	3 added to labels at row end.
0027	C 0008	X 0000	
0028	A 0011	C 0011	
0029	000000	U 0005	Return to subroutine beginning in 0005.
0030	B 0055	L 0058	Maximum allowable error placed in A. $\Sigma\|\delta\|$ placed in L.
0031	000000	T 0039	If error is within limits, control transferred.
0032	B 0049	C 0005	If error is greater than or equal to max. allowable error, averaging continued.
0033	A 0050	C 0006	Original orders reset in 0005, 0006, 0008, 0011.
0034	A 0051	C 0008	
0035	A 0052	C 0011	
0036	C 0056	C 0058	"No. of averages" count set to zero. $\Sigma\|\delta\|$ set to zero.
0037	A 0053	C 0057	$(q - 2) = 28$ placed in A. $(q - 2) = 28$ cleared to 0057.
0038	00000	U 0005	
0039	(530100)	L 0062	
0040	B 0039	Q 0043	
0041	A 0061	C 0039	Final values inserted in tape 3.
0042	000000	U 0039	
0043	000000	4.0000	Machine stopped.

Explanation of Storage

0044	025000	000000	To divide the sum of the values of the four surrounding points by four.

			Explanation of Storage
0045	000001	000001	To alter orders so the same process can be carried out for the next point to the right.
0046	000000	000001	
0047	000003	000003	
0048	000000	000003	
0049	B 0101	A 0130	To put the initial orders in the associated memory positions in preparation for the execution of the next iteration.
0050	A 0132	A 0161	
0051	H 0059	S 0131	
0052	A 0059	C 0131	
0053	000000	000028	Contains $q - 2$ which is used to determine when a row is completed. (Constant)
0054	000000	000784	Contains $(p - 2)\ (q - 2)$ for determining the completion of an iteration. (Constant)
0055			Contains the maximum allowable error against which $\Sigma\|\delta\|$ is checked.
0056	000000	000000	Contains count for number of averages which is checked against 0057 and 0054 to determine the end of the row and end of iteration, range from 0–784. (Variable)
0057	000000	000028	Contains $n(q - 2)$ to determine the end of a row. (Variable)
0058	000000	000000	To store the sum of the absolute difference between the values at points for successive iterations. (Variable)
0059			Used for temporary storage.
0060	320940	A 0001	Used in tape orders.
0061	000060	000000	
0062	530940	L 0062	

Time Required for Solution of the Problem.—The following formula enables one to calculate the number of seconds required for the UNIVAC solution of the problem if the values of p and q are specified and if the number of iterations is known or can be estimated:

Time required in seconds

$$= 2.3 + x[(p - 2)([q - 2].01071 + .005075)] + x(.000945) + (x - 1)(.004235),$$

where x is the number of iterations, (e.g.: $p = 30, q = 30, t = 2.3 + 8.6x$ seconds). For p and $q = 30$, we see that each iteration requires 8.6 seconds. There is also a period of 2.3 seconds associated with input and output, which time is independent of the number of iterations so long as the storage requirements do not exceed the internal memory capacity. This applies, then, to all values of p and q whenever the product pq does not exceed 900.

Estimation of the number of iterations required is not always easy, and no attempt will be made here to discuss this except to point out that this depends not only upon p, q, and the maximum allowable error, but also upon the boundary conditions.

When, to obtain the proper fineness of lattice, p and q must be chosen so large that their product exceeds 900, the external magnetic tape memory has to be used during the course of the problem solution to accommodate some of the information. Since the UNIVAC can be equipped with as many as 12 magnetic tapes, all under its automatic control, and each of these can store approximately one million decimal digits, there is ample capacity for handling problems for which hundreds of thousands of lattice points are used. There is no essential difficulty in programming problems of this mag-

nitude. The programming is particularly easy when either p or q, whichever is smaller, is small enough so that three rows (or columns) of the lattice can be held in the internal memory at one time; that is, problems for which either p or q is less than 300 are easily programmed. If both p and q exceed 300, somewhat more elaborate programming must be used, and more time will have to be allowed for transfer of data to and from tapes.

The formula which has been given here to be used in estimating time of solution is strictly applicable only when the product pq is less than 900. However, it will be observed that the computation time per iteration is almost four times as large as the time required for input and output. (The high external-internal transfer rate of 10 000 decimal digits per second has been provided so that limitations from this cause normally do not occur.) It may be expected that the additional programming required to accomplish the transfers to and from magnetic tape for these larger problems will add slightly to the operating time of the computer, but the above considerations would indicate that one could take $\frac{1}{100}pq$ as the approximate time in seconds for each iteration of a large problem. Since the number of iterations required depends in a complex way upon various factors, including the boundary conditions, and is strongly dependent upon pq, there is no need to have a more exact estimate of the iteration time for a large problem.

Summary.—The solution of the plane potential problem using the Liebmann method of finite differences has been formulated in terms of the instruction code for the UNIVAC computer. Explicit coding has been given only for the simple rectangular boundary, but the nature of the modifications required for more general boundaries has been discussed. Time estimates for each iteration have been made, and other topics such as convergence tests, truncation errors, and scale factors have been considered. Problems with hundreds of thousands of lattice points can be handled automatically by the UNIVAC system using coded routines only slightly more complicated than the one presented.

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E-MCC

EDITORIAL NOTE: It is believed that there is wide interest in the question of instructing high-speed electronic computers now under design to perform the sequences of operations pertinent to selected problems. This article, submitted by members of the staff of a company engaged in the development and construction of electronic digital computers, is considered to be a useful introduction to the use of the instruction code for the computer therein discussed. It should perhaps be pointed out here, however, that the tone of the article is not to be construed to mean that UNIVAC systems have a history of successful operation, rather as an indication of the familiarity of the authors with the design features of the proposed machine and their evaluation of its potential utility. In fact, the construction of the first UNIVAC has not yet been completed.

[1] The UNIVAC trade mark and Instruction Code were in 1948 copyrighted by the Eckert-Mauchly Computer Corporation (E-MCC), Philadelphia, Pa.

[2] The UNIVAC system was designed by the E-MCC under contract with the NBS, supported by the Bureau of Census (BC). The investigation of the suitability of the system was carried out jointly by the NBS and the E-MCC together with the help of interested persons in the BC.

[3] H. LIEBMANN, "Die ausgenährte Ermittelung harmonischer Funktionen und konformer Abbildungen (nach Ideen von Boltzmann and Jacobi)," Akad. d. Wissen., Munich, *Berichte*, 1918, p. 385–416. The Liebmann method is actually a specialization of the GAUSS-SEIDEL iterative process written up by G. SHORTLEY & R. WELLER, "The numerical solution of Laplace's equation," *Jn. Appl. Physics*, v. 9, 1938, p. 334–348.

Complex Zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$

One of the authors has obtained conditions for the existence of real and of complex zeros on the various branches of Bessel functions of real order.[1] For the integral order Bessel functions of the second kind, $Y_n(z)$, there are real zeros only on the branch which is real along the positive real axis. (These zeros are positive.) Every other branch has non-real zeros in the right half-plane and all branches have non-real zeros in the left half-plane.

Below are listed the first fifteen (non-real) zeros of $Y_0(z)$, $Y_1(z) = -Y_0'(z)$, and $Y_1'(z)$ in the left half-plane on the branch which has positive real zeros. Approximations to the first few zeros tabulated were obtained from "contour lines" of $Y_0(z)$ and $Y_1(z)$ which are soon to be published.[2] These approximations were refined by Newton's iteration method. The process was not at all simple since it involved the calculation to maximum accuracy of the Y's and their derivatives at points off the rays for which the functions have been tabulated.

Zeros with absolute values greater than 10 (i.e. those outside the range of the above mentioned volume) were computed from the following asymptotic expansions:[3]

$$-z_j \sim \beta - \frac{4n^2-1}{8\beta} - \frac{112n^4-152n^2+31}{384\beta^3} - \cdots,$$

$$-z_j' \sim \beta_1 - \frac{4n^2+3}{8\beta_1} - \frac{112n^4+328n^2-9}{384\beta_1^3} - \cdots$$

where the z_j are zeros of $Y_n(z)$, the z_j' are zeros of $Y_n'(z)$, $\beta = (j+\frac{1}{2}n-\frac{1}{4})\pi - i \operatorname{arctanh} \frac{1}{2}$, and $\beta_1 = (j+\frac{1}{2}n+\frac{1}{4})\pi - i \operatorname{arctanh} \frac{1}{2}$.

Table A below lists to 9D the zeros with absolute values less than 10. With the zeros of Y_0 are the values of Y_1 and Y_1' at the zeros. Similarly for the zeros of Y_1 and Y_1'. **Table B** has fifteen zeros to 5D for each of Y_0, Y_1, and Y_1'.

Table A

Zeros $z_{0,s}$ of $Y_0(z)$ and Values of Y_1 and Y_1' at the Zero

	$z_{0,s}$		Y_1		Y_1'	
s	Real	Imag.	Real	Imag.	Real	Imag.
1	−2.40301 6632	+.53988 2313	+.10074 7689	−.88196 7710	+.11840 7791	−.34042 2712
2	−5.51987 6702	+.54718 0011	−.02924 6418	+.58716 9503	−.01568 8932	+.10481 8434
3	−8.65367 2403	+.54841 2067	+.01490 8063	−.46945 8752	+.00514 0082	−.05392 3912

Zeros $z_{1,s}$ of $Y_1(z)$ and Values of Y_0 and Y_1' at the Zero

	$z_{1,s}$		$Y_0 = Y_1'$	
s	Real	Imag.	Real	Imag.
1	− .50274 3273	+.78624 3714	−.45952 7684	+1.31710 1937
2	−3.83353 5193	+.56235 6538	+.04830 1909	− .69251 2884
3	−7.01590 3683	+.55339 3046	−.02012 6949	+ .51864 2833

Zeros $z_{1',s}$ of $Y_1'(z)$ and Values of Y_0 and Y_1 at the Zero

	$z_{1',s}$		Y_0		Y_1	
s	Real	Imag.	Real	Imag.	Real	Imag.
1	+ .57678 5129	+.90398 4792	+.08026 5303	+.89580 3699	−.76349 7088	+.58924 4865
2	−1.94047 7342	+.72118 5919	−.23359 1479	+.40380 0129	+.16206 4006	−.95202 7886
3	−5.33347 8617	+.56721 9637	+.01766 2838	−.11002 8559	−.03179 4008	+.59685 3673
4	−8.53676 8577	+.55606 0704	−.00538 9206	+.05500 9625	+.01541 7716	−.47260 1166

Table B

Complex Zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$

	$z_{0,s}$		$z_{1,s}$		$z_{1,s}'$	
s	Real	Imag.	Real	Imag.	Real	Imag.
1	− 2.40302	.53988	− .50274	.78624	+ .57679	.90398
2	− 5.51988	.54718	− 3.83354	.56236	− 1.94048	.72119
3	− 8.65367	.54841	− 7.01590	.55339	− 5.33348	.56722
4	−11.79151	.54882	−10.17358	.55127	− 8.53677	.55606
5	−14.93091	.54900	−13.32374	.55046	−11.70618	.55286
6	−18.07106	.54910	−16.47066	.55006	−14.86367	.55150
7	−21.21163	.54915	−19.61587	.54984	−18.01557	.55080
8	−24.35247	.54919	−22.76009	.54970	−21.16440	.55038
9	−27.49348	.54922	−25.90368	.54961	−24.31135	.55012
10	−30.63461	.54923	−29.04683	.54955	−27.45706	.54995
11	−33.77582	.54925	−32.18968	.54950	−30.60193	.54982
12	−36.91710	.54926	−35.33231	.54947	−33.74619	.54973
13	−40.05843	.54926	−38.47477	.54945	−36.88999	.54966
14	−43.19979	.54927	−41.61710	.54942	−40.03345	.54961
15	−46.34119	.54927	−44.75932	.54941	−43.17663	.54956

ABRAHAM HILLMAN & IVA SHERMAN

NBSCL

[1] A. HILLMAN, "On the reality of zeros of Bessel functions," Amer. Math. Soc., *Bull.*, v. 55, 1949.

[2] NBSCL, *Tables of the Bessel Functions $Y_0(z)$ and $Y_1(z)$ for Complex Arguments*, New York, Columbia University Press, publication announced for 1949.

[3] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. Second ed. Cambridge and New York, 1944, p. 505–507.

A Method of Plotting on Standard IBM Equipment

The advent of automatic computing machinery in research in physics and chemistry has eliminated bulky arithmetic procedures, but in many cases this advantage is lost by the bottleneck of plotting the results; as for example in spectrum analysis where many computed curves have to be compared with the data in the form of a graph. If the results are computed on cards, or are in volume enough to be punched on cards, the following procedure simplifies the problem of plotting a large number of points.

The cards are sorted on abscissa and are fed into an IBM tabulator (preferably a 405) fitted with two digit selectors and preferably six class selectors. Each spacing of the platen (corresponding to each line in a typewritten roll of paper) is taken as unit increase in the abscissa. The range of the abscissa is then infinite—with obvious practical limitations. The unit spacing of the ordinate is that between type bars, of which there are 88 across the paper plus a space between the alphabetical and numerical sections. The range of ordinates is therefore limited to 0 to 88. This, however, is a suitable match for the abscissa scale for a reasonable length of paper. If the numbers to be plotted do not lie between 0 and 88 or lie in only a small fraction of this range, say 0 to 20, they can be machine-multiplied beforehand.

The problem then is, for a given value of the ordinate (punched on the card), say 35, to actuate the thirty-fifth type bar, which prints a symbol

(a digit 1–9) which is the "point" on the plot. This is done, in principle, as follows. The tens digit (3 in the example) is picked up by the upper control brushes as the card enters the machine. This signal is filtered through a digit selector (DSC No. 1). The "3" picks up a class selector No. 3. The units position is picked up by the lower brushes and goes through the other DSC No. 2. In the example the "5" goes to the fifth hub on every class selector. But, as we have seen, only selector No. 3 is picked up. The selectors are wired so that the impulse (in the example, "5") from the lower brush via DSC No. 2 goes through class selector No. 3 to type bar No. 35. A "5" is printed 35 spaces from the left edge of the paper (or suitable base line). The "points" on the plot then are the printed units-place digit. They can be connected by hand or ruler to form a graph. The accuracy is clearly about 0.3 in 88, and is quite adequate for plotting data with two significant figures.

In practice the above scheme is modified, to allow ordinates and abscissas to be printed for reference: to use counters as well as selectors, printing zeros, allowing for the "missing" type bar between alphabetical and numerical sections, etc.

Details: For operators of the 405 Tabulators the following detailed account of the method may be given in the standard IBM terminology. By the above scheme one could print numbers 0 to 88, but it is well worth while to sacrifice the first four alphabetical type bars for printing the abscissa. This also constructs the abscissa scale. It is also worth while to sacrifice the last three type bars to print the ordinate, as it is convenient to have the ordinate printed for later reference or as a check while drawing in the curve. It is also convenient to have three significant figures printed even though they are not plotted. The printing is then as follows:

Alphabetical type bars AT 1–4, abscissa.
AT 5–43, ordinates plotting points 0–38.
Gap between AT 43 and NT 1 corresponds to ordinate 39, which is printed as below.
Numerical type bars NT 1–42, correspond to ordinates 40–81.
NT 43–45, ordinate.

Type bars AT 5 to 43 are operated by the usual class selectors A to D picked up by impulse 0, 1, 2, 3, from DSC No. 1 to their D hubs. (Note that for numbers less than 10 the tens digit, 0, must be punched on the card.) Type bars NT 1 to 20 can be actuated through two other class selectors E and F when they are available, otherwise by counters, as below. The remaining numerical type bars are actuated through counters, e.g., 2D + 8D, 2C + 8C, and 2B. The counters have their plug to C supplied through X distributors No. 1, 2, 3, which in turn are picked up by impulses 6, 7, 8 from DSC No. 1 to their D hubs.

The units digit, picked up from a lower brush, goes to DSC No. 2. The hubs 1 to 9 of this DSC go to positions 2 to 10 in class selector and counter groups. Note that the DSC hubs are double. One set can go to a counter. Entry hubs are also double, so can be wired to all the other counters in succession. The last free set of hubs can then go to a class selector C hub. The other DSC hubs go to another selector. If only single points are plotted from each card, the other selector C hubs can be wired from No. X hubs of the two

already wired in. However, if multiple printing is anticipated, split wires will be needed.

To print zeros at 0, 10, 20, 30, etc., the units position picked up by upper control brushes goes to a comparing magnet. An unequal impulse there picks up X distributor (No. 4, say) which passes on No. X a "hot 9" to the first position on all selectors and counters. Thus a "9" is printed instead of a "0" in these cases. This is slightly confusing since all other plotted "points" are precisely the units-place digits, but this is a very trivial point. As a matter of fact the zero in 0, 10, 20, and 30 can be picked up on the control brush and wired to the zone magnets on the alphabetical type bars. All hammersplit levers are up except for AT 1 to 4 and NT 43 to 45.

The ordinate "39" needs special attention since there is no type bar corresponding to this ordinate. A simple way to take care of this is to print an asterisk in NT 1 instead of the "9" which would appear there, correctly, for ordinate 40. This can be done as follows. The tens digit is taken from the control brushes to one side of a comparing magnet. The other side of this magnet is connected to DSC No. 1 hub 3. Thus an impulse arises in this comparing magnet whenever the tens digit is not "3." The neighboring comparing magnet compares the units position as detected on the control brushes and compares it with a hot 9. Thus this gives a signal unless the units position is a "9." The "unequal" impulse leads of the two comparing magnets are bottle-plugged together, and the connecting hub wired to the pickup hub of a three pole X distributor (say No. 5). One of the common hubs of this distributor receives an "SUP" signal and its No. X hub is wired to NT 1, through No. X and C of another circuit in the distributor. The third circuit receives the "9" for normal printing of "40" in its C hub, which is passed to X, then to X of the second circuit—which, when the distributor is picked up, goes through C hub (of the second circuit) to NT 1. So far, the circuits do not discriminate between 09 and 39. Thus the SUP passes through an X distributor picked up by the "3."

One convenient feature of the method is that plots can be made from numbers punched anywhere on the cards. Only two wires for the upper and lower brushes need be shifted to change fields.

Double and triple spacing can be obtained, thus giving factors of 2 and 3 on the abscissa.

With certain modifications, the method can be used to plot accumulated totals, by card cycle transfer.

The method is also useful in plotting points not necessarily falling on a continuous curve. For example, in a correlation diagram, a number of values of x are plotted against y. If there is no correlation, the points lie at random, and if there is, they cluster in some region or around a 45° line, etc. A large number of such points can be plotted very quickly by this method. Some difficulty arises when there are more than one pair (x, y) to be plotted at the same place, which can however be easily resolved by coding.

The technique of plotting was tested by reproducing the original drawing of the experimental data. The drawing of the data was cut out from a reprint of the original article, projected through an enlarger on graph paper. The ordinate at each abscissa was read off the graph prepared in this way. These numbers were key-punched. The cards were put through the tabulator

with the plotting plug board. The points printed by the tabulator were connected. This forms our graph of the original data. To test the method this curve was followed by a pantograph which reduced it to the size of the original drawing. This small scale plot was compared directly with the original. The agreement was excellent in spite of the number of steps.

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EDITORIAL NOTE: Although the method here described appears to have been original with Mr. KING, W. J. E. informs us that it has been used in several places for several years, but does not seem to have been previously described in print.

RECENT MATHEMATICAL TABLES

578[A].—Schomann's, *(1 × 1 Tabelle) 1–99 × 99 und 1–999 × 9*. Hamburg, Germany, Verlag Br. Sachse, n.d., 16 p. 14.2 × 20.6 cm.

This little paper-covered multiplication table gives, p. 2–9, the results of multiplications of pairs of numbers 1(1)99 and 1(1)99; and, p. 10–15, of pairs 1(1)999 and 1(1)9. The use of the table to find 8379 × 5623 and 8967 × 456 is indicated.

579[A].—H. S. UHLER, "Twenty exact factorials between 304! and 401!," Nat. Acad. Sci., *Proc.*, v. 34, Aug. 1948, p. 407–412. 17.4 × 25.7 cm.

The text: "In the year 1944 the author published privately a little book entitled *Exact Values of the First 200 Factorials*. [See *MTAC*, v. 1, p. 312.] Subsequently he computed with great care the exact values of $n!$ from $n = 201$ to $n = 300$. The data of this third century have not appeared in print. One consultable copy has been deposited in the library of Brown University, Providence, Rhode Island, and another copy is in the possession of Doctor J. C. P. Miller, technical director of Scientific Computing Service Limited, 23 Bedford Square, London, W.C.1, England.

"Recently the author has computed a skeleton table of 42 exact factorials beginning with 303! and ending with 400!. This table was built up by first calculating the values of $n!$ for which $n + 1$ was one of the 17 primes from $n = 307$ to $n = 401$, so that Wilson's theorem could be applied as a more exacting check in addition to congruence testing with moduli such as $10^5 + 1$, $10^8 + 1$, etc. Incidentally the values of 350!, 372!, 375!, 378! and 400! as found by the author in February, 1945, were reproduced identically in the work performed three years later. In order to make a few of these arithmetical constants available to other investigators requiring exact values in the fourth century of $n!$ the following table of equally spaced but non-consecutive data is presented." [Then follows $n!$, $n = 305(5)400$; in the last there are 869 digits.]

EDITORIAL NOTE: Professor UHLER has reported a printers' error under $340!/10^{83}$, in the second line, 11th pentad, which reads 58229 erroneously instead of the correct order 85229. This correction was made in reprints.

580[C, D].—FRANCE, INSTITUT GÉOGRAPHIQUE NATIONAL, *Tables des Logarithmes à Huit Décimales*. Tome 1: *Logarithmes des Nombres entiers de 1 à 120 000;* Tome 2: *Logarithmes des Fonctions Circulaires de dix secondes en dix secondes d'arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1944 [x, 216, x, 402], p. 20.7 × 27 cm. 2700 francs, unbound.

This is the second edition of the great work issued by the Service Géographique de l'Armée in 1891, to which we have had occasion more than once to make reference (*MTAC*, v. 1, p. 36, 85, 145; v. 2, p. 181). The first edition was a single-volume work with pages of

size 27.5 × 34.7 cm. In the offset reproduction the print page has been reduced in size in the ratio 27:22. A new preface is added. Log sin, log cos, log tan, log cot are tabulated for each $0^g.001$. All of the errors we listed on p. 85 have been corrected, but not the single known error of v. 1, noted on p. 181.

R. C. A.

581[C, E].—GEOFFREY BEALL, "The transformation of data from entomological field experiments so that the analysis of variance becomes applicable," *Biometrika*, v. 32, p. 243–262, 1942.

On p. 250–251 there is a table of $k^{-\frac{1}{2}} \sinh^{-1} (kx)^{\frac{1}{2}}$ for $x = [0(1)50(5)100(10)300; 2D]$, $k = 0(.02).1(.05).3(.1).6(.2)1$.

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582[D, P].—CONSIGLIO NAZIONALE DELLE RICERCHE, *Questioni di Matematica Applicata. Trattate nel 1° Convegno di Matematica Applicata (Roma, 1936), da M. Picone, G. Krall, C. Ferrari.* Bologna, Zanichelli, 1939, iv, 155 p. 15 × 23.5 cm. 200 lire. GIULIO KRALL, "Strutture in foglio (a scatola), volte-travi e volte secondo superfici di traslazione. Applicazioni alle costruzioni civili ed idrauliche," p. 37–131.

Tables, p. 57–58, of $\alpha(K, \psi)$ and $\beta(K, \psi)$, $K = 0(.1)1$, $\psi = [0, 7°.5, 15°(15°)90°; 3D]$, where $\alpha = (l + K^2 \cot^2 \psi)^{-\frac{1}{2}}$, $\beta = K(K^2 l + \tan^2 \psi)^{-\frac{1}{2}}$, $K = b/a$, the ratio of semi-axes of an ellipse, and $l = 1$.

Tables, p. 59–68, of t_2, s, t_1, t_2', s', t_1', for $K = .1(.1)1$, $\psi = [0, 7°.5, 15°(15°)90°; 5\text{–}6S]$, where $t_2 = q^3K^{-1} \cos \psi$, $t_2' = q^3K^{-1} \cos^2 \psi$, $s = \sin \psi[2 + 3e^2q^2K^{-2} \cos^2 \psi]$, $t_1 = \cos \psi[2Kq^{-3} + 3e^2K^{-1}q^{-1} (\cos^2 \psi - 2q^2K^{-2} \sin^2 \psi)]$, $s' = \sin 2\psi \,(1 + e^2q^2K^{-2} \cos^2 \psi)$, $t_1' = 3[(Kq^{-3} + e^2 \cos^2 \psi K^{-1}q^{-1}) \cos 2\psi + \frac{1}{3}qs^2K^{-3} \sin 2\psi]$, $e^2 = 1 - K^2$, $q = K(\sin^2 \psi + K^2 \cos^2 \psi)^{-\frac{1}{2}}$.

Tables, p. 82–83, of $f(n, \psi) = (n^2 \cos^2 \psi + \sin^2 \psi)^{\frac{1}{2}}$, for $n = [1(.5)5; 4D]$, $\psi = 0, 7°.5, 15°(15°)90°$, and of $[f(n, \psi)]^{-1}$ to 5D.

Tables, p. 94–97, 99–101, 3D, of

$$s(\phi, \delta) = 2K^{-1} \cos^2 \phi_0 \cos^2 \delta_0 \tan \phi \tan \delta - 2 \cos^2 \delta_0 \sin \phi \tan \delta - 2K^{-1} \cos^2 \phi_0 \tan \phi \sin \delta,$$

and

$$\begin{aligned} t_\phi(\phi, \delta) = {} & 2K^{-2} \cos^2 \phi_0 \cos^2 \delta_0 (\cos \phi - \cos \phi_0)(\cos \phi \cos^2 \delta)^{-1} \\ & + \cos^2 \delta_0 K^{-1} (\sin^2 \phi - \sin^2 \phi_0)(\cos \phi \cos^2 \delta)^{-1} \\ & - 2K^{-2} (\cos \phi)^{-1} \cos^2 \phi_0 \cos \delta \,(\cos \phi - \cos \phi_0) \end{aligned}$$

for (p. 94–95) $\phi_0 = \delta_0 = \frac{1}{4}\pi$, $K = 1$, ϕ and $\delta = 0(.1).7$, $\frac{1}{4}\pi$; for (p. 96–97) $\phi_0 = .487^r = 27°53'6''$, $\delta_0 = \frac{1}{4}\pi$, $K = .8$, $\delta = 0(.1).7, .785$; $\phi = 0(.1).4, .487$; for $s(\phi, \delta)$ and $t_\phi(\phi, \delta)$ (p. 99–101) ϕ_0 and $\delta_0 = 30° = .524^r$, ϕ and $\delta = 0(.1).5, .524$, and ϕ_0 and $\delta_0 = 15° = .262^r$, ϕ and $\delta = 0$ (for t_ϕ only), .1, .2, .262.

Extracts from Text

583[D, P].—WILHELM JORDAN (1842–1899), *Hilfstafeln für Tachymetrie*, Twelfth ed. Stuttgart, Metzlersche Verlagsbuchhandlung, 1939. xvi, 246 p. 15.4 × 22.9 cm.

The first edition of this work was published in 1880, the eighth in 1924, and the ninth in 1928. The twelfth edition is an unchanged reprint of the ninth, which in turn was an unchanged reprint of the eighth.

The first 243 pages are filled with tables of $N \sin \alpha \cos \alpha$ and $N \cos^2 \alpha$, $N = 10(1)250$. Up to $N = 100$, $N \sin \alpha \cos \alpha$ is given for $\alpha = [0(3')29°57'; 2D]$, but $N \cos^2 \alpha$ for

$\alpha = [0(1°)10°(30')20°(20')30°; 1D]$. Then for $N = 100(1)175$, $N \sin\alpha\cos\alpha$ is given for $\alpha = [0(2')19°58'; 2D]$, but $N\cos^2\alpha$ for $\alpha = [0(30')10°(20')20°; 1D]$. For $N = 175(1)250$, $N\sin\alpha\cos\alpha$ is given for $\alpha = [0(1')9°59'; 2D]$, and $N\cos^2\alpha$ for $[0(30')5°(20')10°; 1D]$. On p. 244 is a table of $100\sin\alpha\cos\alpha$ for $\alpha = [0(1')9°59'; 3D]$, and of $100\cos^2\alpha$ for $\alpha = [0(10')10°; 2D]$. On p. 245 is $100\sin^2\alpha$, $\alpha = [0(1')11°59'; 3D]$. On p. 246 is a correction table giving $l = -Ck^{-1} + Dk^{-1}$, for $C = 0$ and .5, $k = 99(.2)101$, $D = [10(10)300; 1D]$. Compare *MTAC*, v. 1, p. 38; v. 3, p. 88, 94.

R. C. A.

584[E, P].—N. I. KARÎAKIN, "Metod uzlovykh deplanatŝiĭ dlîa rascheta tonkostennykh mnogoproletnykh sterzhneĭ na kruchenie" [The method of nodal levelling for the solution of thin multiple-spanned rods under torsion.], *Vestnik Inzhenerov i Tekhnikov*, Moscow, v. 24, May 1948, p. 114–117.

T. 3, p. 116, is a 3D table of η, μ, $\nu = \eta + \mu$, for $u = 0(.1)6$ where

$$\mu = (u^2 - u\sinh u)/[2(\cosh u - 1) - u\sinh u],$$
$$\eta = (u\sinh u - u^2\cosh u)/[2(\cosh u - 1) - u\sinh u].$$

The values of ν for $u = 2.9$ and 3.1 are erroneous.

585[F].—ANON., "List of primes 12855 [=25409] and powers of primes 5954 [=10000] with periods not greater than 100 [=144]." *The Duodecimal Bulletin*, v. 4, Oct. 1948, p. 20–26. 14 × 21.5 cm.

This table is in duodecimal notation and gives the primes and powers of primes, as limited in the title, which divide $a^n - 1$ for $a = 2, 3, 5, 6, 7, 10, 11, 12$ and $n = 1(1)144$. Thus, when written to the base a, the number $1/p$ (where p is a prime or a power of a prime) will be periodic of period n. The table has been "translated" from A. J. C. CUNNINGHAM, H. J. WOODALL, & T. G. CREAK, *Haupt-Exponents, Residue-Indices, Primitive Roots and Standard Congruences*, London, 1922.

D. H. L.

586[F].—ALBERT GLODEN, "Solutions minima de la congruence $X^4 + 1 \equiv 0 \pmod{p^\alpha}$, $\alpha = 2, 3$, ou 4, pour $p < 10^3$," *Euclides*, Madrid, v. 8, 1948, p. 126. 16.6 × 24.1 cm.

This small table is a kind of supplement to the very extensive tables of the solutions of the congruence $X^4 + 1 \equiv 0 \pmod{p}$ reported in *MTAC*, v. 2, p. 71–72, 210–211, 300–301. The title is a little misleading. The actual table is for $\alpha = 2$ and all possible primes $p < 1000$, together with the 4 cases $p^\alpha = 17^3, 17^4, 41^3, 41^4$.

D. H. L.

587[F].—M. KRAITCHIK, "On the divisibility of factorials," *Scripta Math.*, v. 14, Mar. 1948, p. 24–26. 17 × 24.6 cm.

Tables are given of the factors of $n! \pm 1$ and of $P_n \pm 1$ where P_n denotes the product of all primes not exceeding n. The factorization of $n! - 1$ is complete through $n = 21$ while small factors, under 1000, are given up to $n = 40$. For $n! + 1$ the factorization is complete through $n = 22$. Small factors are also given through $n = 40$. $P_n - 1$ and $P_n + 1$ are completely factored through $n = 47$ and 53 respectively. Small factors are given through $n = 89$. The numbers $P_n + 1$, sometimes called Euclidean numbers, are of interest because of Euclid's proof of the infinity of primes according to which these numbers are either primes or products of primes greater than n. The first 5 Euclidean numbers are all primes. These are followed by 5 composite numbers. The 11th number $P_{31} + 1 = 200560490131$ was identi-

fied as a prime by D. N. LEHMER on Sept. 16, 1934. The present table shows that $P_n + 1$ is composite for at least the next seven cases.

D. H. L.

588[I].—HERBERT E. SALZER, "Tables of coefficients for interpolating in functions of two variables," *Jn. Math. Phys.*, v. 26, 1948, p. 294–305. 17.5 × 25.3 cm.

In a previous note[1] the author has shown that the multiple GREGORY-NEWTON interpolation formula of order n can be rewritten in the form of a LAGRANGE-type formula, thus

$$f(x + ph_1, y + qh_2) = \sum_{i+j=0}^{n} A_{i,j} f(x + ih_1, y + jh_2).$$

Here the summation ranges over all combinations i, j for which $i + j \leq n$ and

$$A_{ij} = \binom{n-p-q}{n-i-j}\binom{p}{i}\binom{p}{j}.$$

The present paper contains a new proof of this formula and its generalization to an arbitrary number of independent variables, as well as tables.

The tables are of exact values of the coefficients $A_{i,j}$ for $n = 2,3,4$ (in the paper the coefficients corresponding to $n = 2,3,4$ are denoted by $A_{i,j}$, $B_{i,j}$, and $C_{i,j}$, respectively). Each of the three sets contains 9 smaller tables corresponding to the values $q = .1(.1).9$. In these tables $p = .1(.1).9$ is the argument, and the $A_{i,j}$ the functions. When $n = 2,3,4$ there are, respectively, 6, 10, and 15 columns for the $A_{i,j}$.

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[1] H. E. SALZER, "Note on interpolation for a function of several variables," Amer. Math. Soc., *Bull.*, v. 51, 1945, p. 279–280.

589[J].—R. LIENARD, *Tables Fondamentales à 50 décimales des Sommes* S_n, u_n, Σ_n. Paris, Centre de Documentation Universitaire, 5 Place de la Sorbonne, 1948, ii, 54 p. 21 × 27.3 cm. Compare *MTAC*, v. 1, p. 456–457; v. 2, p. 17, 138–139; v. 3, p. 42.

$$S_n = 1 + 2^{-n} + 3^{-n} + 4^{-n} + \cdots,$$
$$s_n = 1 - 2^{-n} + 3^{-n} - 4^{-n} + 5^{-n} - 6^{-n} + \cdots,$$
$$u_n = 1 - 3^{-n} + 5^{-n} - 7^{-n} + \cdots,$$
$$\Sigma_n = 2^{-n} + 3^{-n} + 5^{-n} + 7^{-n} + \cdots.$$

T. I (p. 15–18) is of S_n, $n = 1(1)167$; **T. II** (p. 19–22): $2^{-n}S_n$, $n = 1(1)167$; **T. III** (p. 23–30): s_n and $1 - s_n$, $n = 1(1)167$; **T. IV** (p. 31–34): $2^{-n}s_n$, $n = 1(1)167$; **T. V** (p. 35–38): U_n, $n = 1(1)105$; **T. VI** (p. 39–46): u_n and $1 - u_n$, $n = 1(1)105$; **T. VII** (p. 47–50): ln S_n, $n = 1(1)167$; **T. VIII** (p. 51–54), Σ_n, $n = 1(1)167$. All of these functions have been tabulated before, but none of them to the extent given by Lienard.

Four of them were first tabulated by EULER: $2^{-n}S_n$ and Σ_n in 1748, S_n in 1755, u_n in 1785. PETERS & STEIN, in *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, p. 90–94, gives 32D values of S_n and $2^{-n}S_n$ for $n = 2(1)100$; $1 - u_n$, for $n = 1(1)53$, and $1 - s_n$, for $n = 2(1)100$. J. W. L. GLAISHER tabulated several of the functions as follows: 32D values of s_n and $1 - s_n$ for $n = 1(1)107$ in 1914, and S_n for $n = 2(1)107$ in 1914; 24D value of Σ_n and ln S_n, $n = 2(2)80$, in 1891; 18D value of u_n, $n = 1(1)38$, in 1912. J. P. GRAM tabulated (1884) ln S_n, to 15D, for $n = 2(1)34$. In H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 2 (1935), published tables of S_n (p. 244), to 32D, $n = 2(1)107$; ln S_n and Σ_n (p. 249–250), to 24D, $n = 2(1)80$; s_n (p. 247–248), to 32D, $n = 1(1)100$; and u_n (p. 304), to 18D, $n = 1(1)38$.

Pages 4–14 are devoted to comments on the methods for calculating the tables.

R. C. A.

590[K].—D. J. Finney, "The Fisher-Yates test of significance in 2×2 contingency tables," *Biometrika*, v. 35, 1948, p. 145–156. 19.1 × 27.2 cm.

Consider a double classification of a population in which each element belongs to one of the classes C_1, C_2 and, at the same time, either to C_1', C_2' (double dichotomy). The observed numbers in a sample can, in an obvious way, be arranged in the form of a 2×2 'contingency table'

$$\begin{matrix} a & A-a \\ b & B-b \end{matrix}$$

where A and B are the totals in the classes C_1 and C_2, respectively, and $a+b$, $A+B-a-b$ the totals in C_1', C_2'. If the two characteristics of classification are statistically independent, then the expected values of the two ratios $(A-a)/a$ and $(B-b)/b$ are the same, but chance fluctuations will produce deviations from this expectation. Various tests of significance of the observed deviations have been proposed, but the matter is still under discussion. The present paper is not intended as a contribution to this controversy, but is devoted mainly to tables facilitating the application of one particular test, proposed by R. A. Fisher. For reasons of symmetry the four entries can be so arranged that $A \geq B$ and $a/A \geq b/B$. Assuming statistical independence, and fixed values of a, A, B', the probability of finding in the left lower corner the particular value b is

$$P_b = \frac{A!B!(a+b)!(A+B-a-b)!}{(A+B)!a!b!(A-a)!(B-b)!}$$

The probability of a deviation as great as or greater than the deviation when the observed number is b is $P_b^* = P_b + P_{b-1} + P_{b-2} + \cdots + P_k$, the summation being continued until $k = a + b - A$ or 0 (whichever is greater). The statistician prescribes an arbitrary significance level p, say $p = .05$, and asks for the largest value b which will make the result significant, that is, the largest integer b satisfying $P_b^* \leq p$. In the tables under review this value b is tabulated for all permissible combinations a,A,B, with $A \leq 15$, $B \leq 15$, and for the significance levels .05, .025, .01, .005, thus permitting either one-sided or two-sided tests. In addition, the value P_b^* is given to three places.

These tables were constructed to solve the same class of problems as the corresponding tables of Fisher & Yates (*MTAC*, v. 1, p. 316–320); however, they possess the advantage of requiring no auxiliary computations as required in the latter tables. The Fisher-Yates tables are based on an approximation whereas these tables are exact to the accuracy recorded; however the range of applicability of these tables is considerably smaller.

Will Feller

591[K].—D. J. Finney & W. L. Stevens, "A table for the calculation of working probits and weights in probit analysis," *Biometrika*, v. 35, 1948, p. 191–201. 19.1 × 27.2 cm.

A variety of statistical estimation problems (in particular in dose-mortality studies, detonation of explosives, etc.) require rather heavy algebra which can be considerably simplified by the use of a change of variables called the probit transformation. The main advantage of the latter is that it reduces the estimation problem to a problem of a more familiar kind, namely, linear regression theory. It is impossible to describe the statistical and computational problems in a short space. The mathematical definitions are as follows. Let P, $0 < P < 1$, be considered as the independent variable, and $Q = 1 - P$. Then the probit Y is a function of P defined by $P = \Phi(Y - 5)$, where $\Phi(x)$ is the normal distribution function

$$\Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt.$$

Four auxiliary functions are required for the estimation technique. The *range*, usually denoted by $1/Z$, is defined by dY/dP. The *maximum* and *minimum working probits* are defined by $Y_{max} = Y + Q/Z$, $Y_{min} = Y - P/Z$. Finally, the weighting coefficient is Z^2/PQ. The FISHER & YATES tables (*MTAC*, v. 1, p. 316–320) give the necessary quantities to carry out this estimation technique.

The tables under review give these same quantities correct to four places, which is the accuracy employed in the Fisher & Yates tables, with the exception of the five-place accuracy employed by them in the weighting coefficient; however, the argument Y is tabulated to hundredths rather than to tenths as in the Fisher & Yates tables. The tables also include the minimum working probit, $Y - P/Z$, which may be convenient at times but which is not essential to the technique.

WILL FELLER

592[K].—R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research*. Edinburgh, Oliver and Boyd. Third ed., rev. and enl., 1948, 112 p. 21.5 × 28 cm. 16 shillings. Compare *MTAC*, v. 1, p. 316–320.

Relatively few changes have been made from earlier editions. To the tables of the 20, 5, 1 and 0.1 percent significance levels of z and the variance ratio $F = e^{2z}$ that appear in previous editions, the authors have added tables of the 10 percent levels. Both 10 percent tables have been published previously (in a slightly different form), that of z by PANSE & AYACHIT (*Indian Jn. Agr. Sci.*, v. 14, p. 244–247, 1944) and that of e^{2z} by MERRINGTON & THOMPSON (*Biometrika*, v. 33, p. 73–88, 1943; see *MTAC*, v. 1, p. 78–79).

The table of the orthogonal polynomials $\xi_r'(x)$, where $x = 1(1)n$, $r = 1(1)5$, has been extended from $n = 3(1)52$ to $n = 3(1)75$. These functions are polynomials in x of degree r, and satisfy the relations

$$\sum_{x=1}^{n} \xi_i'(x)\xi_j'(x) = 0, \qquad i \neq j.$$

Further, each function is multiplied by the smallest factor that makes all values of $\xi_i'(x)$ integral. The functions greatly facilitate the work of fitting a polynomial by least squares to a set of data $y(x)$ recorded at equally-spaced intervals in x. Similar tables over the range $n = 1(1)104$ have been published by R. L. ANDERSON & E. E. HOUSEMAN, Iowa Agr. Exp. Sta., *Res. Bull.*, 297, 1942; see *MTAC*, v. 1, p. 148–150.

An unspectacular addition which is likely to be appreciated by statisticians is the inclusion of a table of natural logs (5D) of numbers in the range .100(.001).999. Many statistical operations necessitate taking the natural log of a probability and for this purpose the authors' previous tables, for arguments between 1 and 100, were unsuitable.

The two remaining new tables were constructed for rather specialized statistical computations and will be described only in general terms. The first, due to FINNEY, gives a series of weighting coefficients for dosage-mortality experiments where there is an appreciable natural mortality among the animals or insects that receive no toxic agent. This makes it necessary to adjust the observed mortalities of the groups that receive the toxic agent. The adjusted death rates are then transformed to a scale (probits) on which they may be expected to bear a linear relation to the log dose. The two constants that define the line are estimated by fitting a weighted linear regression, using the weights developed by Finney.

The second table (due to Fisher) gives a series of "scores" that are used very ingeniously in estimating the amount of linkage by maximum likelihood, from the progeny of crosses between double heterozygotes.

Progress continues to be made in the authors' catalogue of balanced incomplete block designs that require ten or fewer replications. The second edition listed 12 cases where the existence of a design had neither been proved nor disproved. The unsolved cases are now reduced to 5. Solutions have been found for $t = 16$, $k = 6$; $t = 21$, $k = 7$; $t = 25$, $k = 9$;

and $t = 31$, $k = 10$, while three cases, $t = 15$, $k = 5$; $t = 22$, $k = 7$; $t = 29$, $k = 8$, have been shown to be impossible. (t = number of treatments, k = number of units per block.)

Minor changes in the presentation of some of the other tables have been made, while parts of the introduction have been re-written.

W. G. COCHRAN

593[K].—JAPAN, HYDROGRAPHIC DEPARTMENT [*Interpolation Tables* 1 *and* 2], Tokyo, Dec. 1946 and Nov. 1947. 94 p. and 107 p., 18.0 × 25.5 cm. In Japanese; printed for home use only and not available for sale.

These tables have been prepared by the Japanese Hydrographic Department to facilitate the sub-tabulation which forms such a large part of the calculation of navigational ephemerides. They clearly owe much to the inspiration of the (British) *Nautical Almanac*, particularly to the section on interpolation (by L. J. COMRIE) in the 1937 edition, reprinted as *Interpolation and Allied Tables*. The notation used is the same and several tables have almost certainly been copied directly. It is, however, the new tables that possess the greatest interest.

All concerned with sub-tabulation on a large scale must, at some time, have made or have contemplated making tables to give directly the end-figures of the interpolates or their differences; the present volumes contain systematic tables for determining the first differences of the interpolates for a variety of intervals. The basis of the method is the splitting of the first and second difference contributions into exact and remainder parts; the various remainders, including where necessary the third difference contributions, are then combined by means of special double- (or triple-) entry tables. There is thus considerable similarity with E. W. BROWN's (*Tables of the Motion of the Moon*, 1919) tables for interpolation to twelfths and some with Comrie's end-figure method of sub-tabulation (see the *Nautical Almanac* for 1931). With the availability of adding machines of large capacity (the Burroughs, National, punched card, relay and electronic machines) essentially simpler and more powerful methods of continuous sub-tabulation can efficiently be used; but these methods involve the use of extra figures which make them unduly laborious for hand calculation. The present tables will be of considerable value where such machines are not available.

The first volume of tables is concerned primarily with interpolation in which third differences (using the Besselian interpolation formula) can be ignored. Tables are given for sub-tabulation to halves, thirds, fourths, fifths, sixths, eighths, tenths and twelfths. Each table consists of two parts. The first gives, under the symbol A, the exact first differences of the second-difference contribution corresponding to chosen values of the double second difference; the range is to about 40,000 and the interval usually the minimum possible (e.g. 400 for tenths and 576 for twelfths), though it is deliberately increased for the larger sub-intervals (e.g. 464 for halves and 306 for thirds) to assist pagination. The second table is double-entry with arguments: the remainder, r, of the first difference divided by the number of intervals, n; the difference, at intervals of 5, between the actual double second difference and the tabular value used in the first table. It gives, under the symbol B, the first difference of the residual first and second difference corrections. The signs of the various contributions are given for positive values of the arguments; other combinations are easily obtained.

Sub-tabulations to $n$ths is thus performed by the following process: put the first difference equal to $qn + r$; take the series of values A from the first table with the nearest tabular value of the double-second difference; take the series of values B from the second table with the remainder r and the residual double-second difference; the series of first differences is the sum of q, A and B; the interpolates are then built up from this series to reproduce the pivotal value as a check on the arithmetic.

Other tables in the volume are straightforward tables giving (i) 4D critical tables of B^{ii} and B^{iii}; (ii) B^{ii}, B^{iii} (7D), B^{iv}, B^{v} (6D) and B^{vi} (5D) at interval .001. A short explana-

tion gives illustrations of the use of both methods, formulae for the throw-back and limits for the neglect of various differences.

The second volume extends the method of the first to include third differences. This requires the use of a triple-entry table which severely limits the scope; the tables are, in fact, restricted to interpolation to fourths, fifths and sixths. In each section there are tables giving series of first differences: A, exact values from second differences; B, exact values from third differences; C, values corresponding to the remainders of the first, second and third differences. The intervals and ranges in the various tables are:

A, double-second difference: $n = 4$, 64, 32256; $n = 5$, 50, 30000; $n = 6$, 144, 20736

B, third difference: $n = 4$, 128, 16896; $n = 5$, 250, 30000; $n = 6$, 648, 23328

C, double-second difference 5 for all n, third difference 10 for $n = 4$, 20 for $n = 5$ and 6. The third table for $n = 6$ occupies 66 pages.

The use of the tables follows that of the first volume, with the exception that modified second and third differences can be used, in the usual way, to extend the method to cover fourth, fifth and sixth differences up to the usual limits. Here the first differences of the interpolates are the sums of q, A, B, C and these are built up to form the pivotal value as a check.

The errors of the method (apart from the use of the throw-back) are solely due to taking the nearest residual values of the second and third differences in forming the series C; they thus reach the maximum of .125 (due to the second difference) and .08 (due to the third difference). These are satisfactorily small and might be considered unduly so in view of the fact that the pivotal values can be in error by .5.

The only other tables in the volume are standard critical tables of the throw-back coefficients .184 and .108. There is, however, a short explanation with adequate examples.

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594[K].—D. Mainland, "Statistical methods in medical research. 1. Qualitative statistics (enumeration data)," *Canadian Jn. Research*, v. 26, 1948, p. 1–166. 17.1 × 25.5 cm.

The purpose of this monograph is to illustrate for workers in medical research the principal statistical techniques that are applicable to data expressed as fractions or percentages. The publication contains a number of tables which although not essentially new are more extensive than those already available and may be useful in many fields in which statistics is applied.

Table I gives confidence limits derived from a single binomial ratio. If a number A in a sample of N have a certain characteristic, the lower and upper confidence limits (f_L, f_u) for the fraction in the population that have this characteristic are defined by the following equations.

$$\sum_{r=A}^{N} \binom{N}{r} f_L{}^r(1 - f_L)^{N-r} = P : \sum_{r=0}^{A} \binom{N}{r} f_u{}^r(1 - f_u)^{N-r} = \mathrm{P},$$

where P is the confidence probability. The ranges are $A = 1(1)20$; $P = .10$, .025, .005. The range of N is irregular; it is approximately $N = 2A(1)2A + 20$, and thereafter by increasingly wide intervals up to $N = 1000$. The limits of f are given as percentages (1D), though strict accuracy to this extent is not claimed. Supplementary tables II and III cover the "large sample" case when A exceeds 20. These tables were computed by means of Table VIII$_1$ in Fisher & Yates' *Statistical Tables* (RMT **593**): that is, by an approximate rather than an exact method. Over 100 entries were checked by exact computation.

Table IV is constructed for the comparison of binomial ratios from two independent samples of equal size N. The data are as follows:

	Sample		
Class	1	2	
C	a	c	$a+c$
not C	b	d	$b+d$
	N	N	$2N$

The expression

$$\frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{(2N)!\,a!\,b!\,c!\,d!}$$

is the probability that this table be obtained, given that (i) the probability of an observation falling in class C is the same in both samples and (ii) the marginal totals in the table are fixed. For $N = 1(1)20$, Table IV gives (4D) the significance probabilities for Fisher's "exact" test of significance in a 2×2 contingency table (*Statistical Methods for Research Workers*, eighth ed., 1941, §21.02), obtained by adding the probabilities given above. Thus if the first sample contains 3 successes and 9 failures, while the second has 8 successes and 4 failures, the probability tabulated (.0498) is the sum of the individual probabilities for the tables.

3	8
9	4

2	9
10	3

1	10
11	2

0	11
12	1

Table V lists all 2×2 contingency tables that reject the null hypothesis that the probability of a C is the same in both samples. The significance levels presented are the 2.5 and 0.5 percent levels, and the exact significance probability for each table is shown. The tables cover any pair of sample sizes (equal or unequal) up to 20. The tables were constructed by addition from the exact probabilities as given in the preceding paragraph.

The remaining tables are of a standard type and need not be discussed.

W. G. Cochran

595[K].—K. R. Nair, "The Studentized form of the extreme mean square test in the analysis of variance," *Biometrika*, v. 35, 1948, p. 16–31. 19.3 × 27.3 cm.

Suppose $\chi_1^2, \chi_2^2, \cdots, \chi_k^2$ are k independent values drawn from a chi-square distribution with m degrees of freedom and arranged in ascending order of magnitude. Let χ_0^2 be a value drawn from a chi-square distribution with ν degrees of freedom and which is independent of the $\chi_1^2, \chi_2^2, \cdots, \chi_k^2$. Let ${}_\nu P_k(Q)$ be the probability that $(\nu\chi_k^2)/(m\chi_0^2) \le Q$.

(In statistical language ${}_\nu P_k(Q)$ is the probability that $(\nu\chi_k^2)/(m\chi_0^2)$, the largest of the k Snedecor F-ratios $(\nu\chi_1^2)/(m\chi_0^2)$, $(\nu\chi_2^2)/(m\chi_0^2)$, $\cdots$, $(\nu\chi_k^2)/(m\chi_0^2)$, will not exceed Q.)

The expression for ${}_\nu P_k(Q)$ is of the form

$${}_\nu P_k(Q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} P_k(Qt)\,dt$$

where

$$P_k(Qt) = \left[\frac{2(\frac{1}{2}m)^{\frac{1}{2}m}}{\Gamma(\frac{1}{2}m)} \int_0^{Qt} x^{m-1} e^{-\frac{1}{2}mx^2}\,dx\right]^k.$$

For $m = 1$,

$$(1)\qquad {}_\nu P_k(Q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} \left(\sqrt{\frac{2}{\pi}} \int_0^{Qt} e^{-\frac{1}{2}x^2} dx \right)^k dt.$$

For $m = 2$,

$${}_\nu P_k(Q) = \sum_{r=0}^{k} (-1)^r \binom{k}{r} \left(1 + \frac{2rQ^2}{\nu} \right)^{-\frac{1}{2}\nu}.$$

Finney (*Annals of Eugenics*, v. 11, 1941, p. 47) has tabulated, for $m = 2$, the values of Q for which ${}_\nu P_k(Q) = .95$ for $\nu = 1(1)10, 20, \infty$ and $k = 1(.1)3$.

Nair deals with tabulations only for $m = 1$. He develops an approximation to (1), making use of an expansion of ${}_\nu P_k(Q)$ in powers of $(1/\nu)$. Using terms up to and including terms of order $(1/\nu)^2$, Nair tabulates approximate values of Q for which ${}_\nu P_k(Q) = 0.95$ and 0.99 (i.e. 1% and 5% significance points of Q) for $k = 1(1)10$ and $\nu = 10,12,15,20,30, 60, \infty$. The author does not give much information on how close the approximations are.

Now let ${}_\nu P_k(q)$ be the probability that $(\nu\chi_1^2)/(m\chi_0^2) \geq q$. (In statistical language, ${}_\nu P_k(q)$ is the probability that $(\nu\chi_1^2)/(m\chi_0^2)$, the smallest of the k Snedecor F-ratios mentioned earlier, will exceed q.) The expression for ${}_\nu P_k(q)$ is of the form

$${}_\nu P_k(q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} P_k(qt) dt,$$

where

$$P_k(qt) = \left[\frac{2(\frac{1}{2}m)^{\frac{1}{2}m}}{\Gamma(\frac{1}{2}m)} \int_{qt}^\infty x^{m-1} e^{-\frac{1}{2}mx^2} dx \right]^k.$$

For $m = 1$,

$${}_\nu P_k(q) = \frac{2(\frac{1}{2}\nu)^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}\nu)} \int_0^\infty t^{\nu-1} e^{-\frac{1}{2}\nu t^2} \left(\sqrt{\frac{2}{\pi}} \int_{qt}^\infty e^{-\frac{1}{2}x^2} dx \right)^k dt.$$

For $m = 2$,

$${}_\nu P_k(q) = \left(1 + \frac{2kq^2}{\nu} \right)^{-\frac{1}{2}\nu},$$

from which 1% and 5% points can be readily computed, although no table is given by Nair or by FINNEY (*loc. cit.*).

For $m = 1$, Nair expands ${}_\nu P_k(q)$ into a power series in $(1/\nu)$ and obtains

$${}_\nu P_k(q) = 1 - \left\{ 1 - a_0 - \frac{a_1}{\nu} \left(1 - \frac{1}{8\nu} \right) \right\}$$

where

$$a_0 = \left((2/\pi)^{\frac{1}{2}} \int_q^\infty e^{-\frac{1}{2}x^2} dx \right)^k,$$

$$a_1 = k(a_0)^{(k-2)/k} \{ (k-1)qz + \tfrac{1}{2}(q^2+1)a_0^{1/k} \},$$

$$z = (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}x^2}.$$

Nair gives a table of values of a_0 and a_1 for $q = 0(.01).10$ and for $k = 2(1)10$. This table can then be used for determining for $k = 2(1)10$ and any value of ν the approximate value of q for which ${}_\nu P_k(q) = .90(.01).99$ (i.e. the 1(1)9 and 10% significance points of ν). The author actually gives a table of 1% and 10% significance points of q for $k = 1(1)10$ and $\nu = 10$.

S. S. W.

596[K, L, M].—FRITZ EMDE, *Jahnke-Emde Tables of Higher Functions. Treated by Fritz Emde. Fourth (revised) edition with 177 figures.* Leipzig, Teubner, 1948, xii, 300 p. There is also a German title page. 16 × 24 cm. See *MTAC*, v. 1, p. 106–109, 161, 198, 202, 204, 293–294, 386, 391–399; v. 2, p. 26, 47, 224, 350; v. 3, p. 254–255, 267–268, 314–315.

The first German edition of this famous work appeared in 1909. Of this first edition there were two corrected reprints in 1923 and 1928. Since JAHNKE died in 1921 all later editions in

Germany were prepared by Professor EMDE. The second (revised) edition, 1933, contained xviii, 330 p. and was bilingual, English and German. The third (revised) edition appeared in 1938, xii, 305 p. and 181 figures; the 76 pages of the second edition devoted to elementary functions are here eliminated. Thus there have been nine editions or reprints of this work.

The present edition contains almost exactly the same number of pages as the third (fifth) since the deletions and additions almost counterbalance. Most of Professor Emde's preface, dated "Pretzfeld, January, 1948," is as follows:

"The Fourth Edition of the Tables of Higher Functions should have been issued in 1944. But after having been printed all copies were destroyed at the book-bindery by bombs and fire during the war. It is only now possible to reprint this edition from the same manuscript.

"As new matter this edition offers:

"1. an extension of the table of the error integral.

"2. the table of the functions of the parabolic cylinder, computed by J. B. Russel [*sic*] (*Journ. of math. and phys.* XII, 1932/3, p. 291–297), checked and corrected by S. Kerridge.

"3. the table of the Laguerre functions, computed by F. Tricomi (Atti R. Acc. Sc. Torino 76, 1941).

"4. the table of the spherical harmonics of the second kind, computed by F. Vandrey (Z. a. M. M. 20, 1940, S. 277–279).

"5. the tables of the incomplete Anger and Weber functions, computed by P. and E. Brauer (Z. a. M. M. 21, 1941, S. 180, 181).

"6. a table of the Bessel functions $J_{n/3}(iy)$ and $H^{(1)}_{n/3}(iy)$ ($n = 1,2$), computed by S. Kerridge, instead of the earlier (incorrect) table of $J_{n/3}(iy)$ and $J_{-n/3}(iy)$, computed by Dinnik. If y is large these two last functions become nearly equal, thus do not represent two linearly independent solutions of the Bessel differential equation.

"7. formulae and figures for the use of Debye's convergently beginning series for the Bessel functions with complex argument and order.

"8. a list of 7 and more place logarithmic tables."

A number of the tabular changes have thus been suggested. Quite a few corrections in the tables have been made but many errors still remain.

The error integral table of $\Phi(x)$ has been extended from a 4D to a 5D table, for $x = 0(.01)3.09$, and a table of $e^{x^2}[1 - \Phi(x)]$, for $x = [3(.01)5; 5D]$ has been added. The former tables and graphs of the derivatives of the error integral have been dropped, and in their place Russell's tables of HERMITE (parabolic cylinder) functions, $\phi_n(x)$, $n = 1(1)11$, $x = [0(.04)1(.1)3.5; 5D]$ have been substituted; see *MTAC*, v. 1, p. 4, 152–153.

These are followed (p. 32–33) by TRICOMI's table of LAGUERRE functions ln (x) $= e^{\frac{1}{2}x}(n!)^{-1}d^n(e^{-x}x^n)/dx^n$, $x = [.1(.1)1(.25)3(.5)6(1)14(2)34; 4D]$, $n = 1(1)10$; see *MTAC*, v. 2, p. 267. The errors which we noted in the $C(u)$ and $S(u)$ tables have been corrected.

In the section on elliptic functions the four pages of tables and graphs for the WEIERSTRASS functions in the equianharmonic case have been eliminated. There are 64 errors in the table of $\log q$ (p. 50); 16 errors in the K, E, θ table; and 7 in the $E(\theta, \phi)$, $F(\theta, \phi)$ tables (A. FLETCHER); also the right-hand member of the equation, p. 80, l. 5, is entirely wrong.

In the section on LEGENDRE functions VANDREY's table of spherical functions of the second kind, $Q_n(x)$, has been added, $x = [0(.01)1; 5D]$, $n = 1(1)7$. The four serious errors in this table which we noted, *MTAC*, v. 1, p. 446, are still in evidence. The quite erroneous figure for $P_\nu(x)$ in the neighborhood of $x = -1$ (which we noted *MTAC*, v. 1, p. 395, 398) has been replaced by an entirely new graph on p. 105 (108). The 53 errors corrected in the tables on p. 124–125 of the latest American ed. are all to be found on p. 120–121 of the work under review.

The greatest change in any section is in that devoted to Bessel functions, p. 125–264 (1945 Ed. p. 126–268). The sections on asymptotic representations and differential equations that give Bessel functions, have been elaborated and the section on integral representations deleted. The 6 pages of tables of the Struve functions S_0, S_1 were eliminated; the 3 pages

devoted to S_0 were no great loss since they duplicated the table given for Ω_0 (with 40 unit errors not in S_0). The following two new BRAUER tables (which we listed *MTAC*, v. 1, p. 245, 282) are given (p. 218–219): Incomplete ANGER function, $\frac{1}{2}\int_0^x \cos \frac{1}{2}q[\frac{1}{2}\pi t - \sin(\frac{1}{2}\pi t)]dt$, $x = 0(.1)2$, $q = [.1(.1)1; 4\text{–}5D]$, and incomplete WEBER function, $\frac{1}{2}\int_0^x \sin \frac{1}{2}q[\frac{1}{2}\pi t - \sin(\frac{1}{2}\pi t)]dt$, for the same ranges, but mostly 5–6D.

The highly erroneous Dinnik tables involving $J_{\pm\frac{1}{2}n}(ix)$, $n = 1,2$, p. 235 have been replaced by Kerridge's full-page, 231, of tables involving $J_{\frac{1}{2}n}$ and $H^{(1)}_{\frac{1}{2}n}(ix)$, $n = 1,2$, $x = [0(.1)10; 4\text{–}5S]$.

Among the errors in this Bessel function section, already noted in *MTAC*, are the following (the first page given being that of the v. under review, the second that of the 1945 edition):

P. 125 (126).—Relief 66 (67), $J_p(x)$ is incorrect near the origin; see *MTAC*, v. 3, p. 315.
P. 164 (164).—$J_{\pm\frac{1}{2}n}(x)$, $n = 1,3$; only 6 of the 32 errors have been corrected.
P. 166 (166).—The error $J_1(x_{17})$ remains.
P. 168 (168).—$J_p(x_n)$, of the 27 errors in this table only the 7 most serious ones have been corrected.
P. 183 (183).—$\Delta_3(6.4)$, diff. incorrect.
P. 204–205 (204–205).—Zeros of $J_p(x)N_p(kx) - J_p(kx)N_p(x)$, still three errors. Compare RMT 566.
P. 224–225 (228–229).—$I_0(x)$, $I_1(x)$, more serious errors have been eliminated but many last figure unit errors remain.
P. 230 (234).—Zeros of $J_n(ix)J_n'(x) - iJ_n(x)J_n'(ix)$, there are still 8 errors.
P. 232 (236).—$iH_0^{(1)}(i) = 3.006$ omitted.
P. 242–243 (246–247).—Re $J_0(x\sqrt{i})$, Im$J_0(x\sqrt{i})$, three errors.

The Section IX on RIEMANN zeta function, p. 265–270 (269–274), is practically unchanged, but under the table of the first 29 zeros of the function, $\sigma + it$, p. 270, references are given to papers by E. C. TITCHMARSH who shows, with tables, that between $t = 0$ and $t = 1468$, there are exactly 1041 zeros all on the line $\sigma = .5$; see R. Soc. London, *Proc.*, v. 151, 1935, p. 234–255, and v. 157, 1936, p. 261–263.

There is practically no change in the two final sections on confluent hypergeometric functions and MATHIEU functions.

In the concluding bibliography, two errors are made in an entry on p. 295, where it is stated that the second ed. of THOMPSON's *Table of the Coefficients of Everett's Central-Difference Interpolation Formula*, was published in 1937 and contained xvi, 20 p. (see *MTAC*, v. 1, p. 185).

The board-binding and canvass back are only fairly substantial and many of the pages are apparently reproduced by the offset process from the corresponding pages of the third edition, or of a surviving volume of the 1944 fourth edition.

R. C. A.

597[L].—PIERO GIORGIO BORDONI, "Sulle funzioni di Stokes," Pontificia Academia Scientiarum, *Commentationes*, v. 9, no. 3, 1945, p.87–113. 17 × 24.5 cm.

The functions here tabulated (p. 104–112), and called respectively Stokes' functions of the first, second and third species, are as follows:

$$f_m(ix) = i^{-(m+1)}e^{ix}(\tfrac{1}{2}\pi x)^{\frac{1}{2}}[J_{m+\frac{1}{2}}(x) + i(-1)^m J_{-m-\frac{1}{2}}(x)]$$
$$= i^{-(m+1)}e^{ix}(\tfrac{1}{2}\pi x)^{\frac{1}{2}}H^1_{m+\frac{1}{2}}(x) = {}_2F_0(-m, m+1, \tfrac{1}{2}x^{-1});$$
$$F_m(ix) = ix(2m+1)^{-1}[mf_{m-1}(ix) + (m+1)f_{m+1}(ix)];$$

$\zeta_m(ix) = f_m(ix)/F_m(ix)$. The tabulation of real and imaginary parts is for $m = 1(1)5$, $x = .1(.05).2(.1)1(.25)4(.5)6(1)10(5)20, 30$. The tabulation is usually 4–5S. There are graphs illustrating the variations in connection with each of these functions.

The six functions $f_m(ix)$, $m = 0(1)5$, are as follows:

$f_0(ix) = 1$; $f_1(ix) = 1 - ix^{-1}$; $f_2(ix) = 1 - 3x^{-2} - 3ix^{-1}$;
$f_3(ix) = 1 - 15x^{-2} - i(6x^{-1} - 15x^{-3})$; $f_4(ix) = 1 - 45x^{-2} + 105x^{-4} - i(10x^{-1} - 105x^{-3})$;
$f_5(ix) = 1 - 105x^{-2} + 945x^{-4} - i(15x^{-1} - 420x^{-3} + 945x^{-5})$.

R. C. A.

598[L].—Great Britain, Admiralty Computing Service, *Lateral Vibration of Beams of Conical Section*. No. SRE/ACS 92, August 1945, 5 p. (4 leaves with cover). 20.3 × 32.2 cm. This publication is available only to certain agencies and activities. Compare *MTAC*, v. 2, p. 289–297.

The differential equation $\frac{d^2}{ds^2}\left(s^4 \frac{d^2y}{ds^2}\right) = s^2y$ has the solution, in terms of Bessel functions,

$$y = s^{-1}[AJ_2(2s^{\frac{1}{2}}) + BY_2(2s^{\frac{1}{2}}) + CI_2(2s^{\frac{1}{2}}) + DK_2(2s^{\frac{1}{2}})]$$

where

$$A/Y_2(\alpha) = -B/J_2(\alpha) = C/[rK_2(\alpha)] = D/[rI_2(\alpha)],$$

$$r = [Y_2(\alpha)J_2(\beta) - J_2(\alpha)Y_2(\beta)]/[I_2(\alpha)K_2(\beta) - K_2(\alpha)I_2(\beta)], \qquad \alpha = 2\sqrt{L}, \qquad \beta = 2\sqrt{aL}.$$

T.I gives, for $a = 0(.1).6$, 4–6S (one 7S), values of $L, A, B, 10^5C, D$.

T.II gives 1–5S values of y, unit 10^{-4}, for $a = 0(.1).6$ and for 11 values of s from aL linear scale to L.

T.III gives 3–4S values of d^2y/ds^2, unit 10^{-6}, for the same values of a and s, as in T.II.

Reference: J. W. Nicholson, "The lateral vibration of bars of variable section," R. Soc. London, *Proc.*, v. 93, 1917, p. 506–519.

Extracts from text

599[L].—Harvard University, Computation Laboratory, *Annals*, v. 10, *Tables of the Bessel Functions of the First Kind of Orders Twenty-Eight through Thirty-Nine*. By the Staff of the Computation Laboratory, Professor H. H. Aiken, director. Cambridge, Mass., Harvard University Press, 1948, x, 694 p. 19.5 × 26.7 cm. $10.00. Offset print. Compare *MTAC*, v. 2, p. 176f, 261f; v. 3, p. 102, 117–118, 185–186.

This is the eighth of the thirteen planned volumes of the great edition of tables of Bessel Functions of the first kind prepared by Harvard's IBM Automatic Sequence Controlled Calculator. The offset print of the splendidly clear copy furnished from this machine is notable. The volume contains tables of $J_n(x)$, $n = 28(1)39$, $x = [0(.01)99.99; 10D]$. The first significant value .00000 00001 of the twelve functions are for $J_{28}(10)$, $J_{29}(10.65)$, $J_{30}(11.32)$, $J_{31}(11.99)$, $J_{32}(12.68)$, $J_{33}(13.37)$, $J_{34}(14.06)$, $J_{35}(14.77)$, $J_{36}(15.48)$, $J_{37}(16.2)$, $J_{38}(16.92)$, $J_{39}(17.65)$.

Practically all of the tens of thousands of entries in this table are new. The only previous duplicating values previously published, to at least 10D, are 50 values in Cambi (1948, see RMT 535) to 11D, and 24 given by Hayashi (1930), for $n = 28(1)39$, $x = 20, 30$, and to at least 31D. In every case the rounded Cambi and Hayashi entries agree with those in the Harvard volume.

As in the case of the seventh volume the computation of the tables in the volume under review, and the preparation of the manuscript, were under the supervision of John A. Harr.

R. C. A.

600[L].—K. A. Kitover, "Tablitsy summ nekotorykh beskonechnykh trigonometricheskikh riadov" [Tables of the sums of certain infinite trigonometric series], Akad. Nauk SSSR *Prikl. Mat. Mekh.*, v. 12, 1948, p. 233–240. 16.2 × 26.1 cm.

The sums referred to in the title are

$$F_{2\nu}(x) = \sum_{n=1}^{\infty} n^{-2\nu} \sin nx, \qquad F_{2\nu+1}(x) = \sum_{n=1}^{\infty} n^{-2\nu-1} \cos nx$$

$$f_{2\nu}(x) = \sum_{n=1}^{\infty} (2n-1)^{-2\nu} \sin (2n-1)x$$

$$f_{2\nu+1}(x) = \sum_{n=1}^{\infty} (2n-1)^{-2\nu-1} \cos (2n-1)x.$$

F_k and f_k are tabulated to 4D for $k = 1(1)6$, and for $x = m\pi/180$, $m = 0(1)180$, that is for x in degrees. The corresponding radian values of x are also given. Because $f_k(\pi - x) = f_k(x)$ the table of f_k extends only as far as $x = \frac{1}{2}\pi$ with a pair of columns for $\pi - x$. The functions F_k and f_k are related by

$$f_k(x) = F_k(x) - 2^{-k}F_k(2x).$$

Seven other similar trigonometric sums are expressed in terms of F_k. Although the functions bear a strong superficial resemblance to the Bernoulli functions which are merely polynomials in the unit interval, F_k and f_k are non-elementary functions, with the exception of

$$F_1(x) = -\ln (2 \sin \tfrac{1}{2}x), \qquad f_1(x) = \ln \cot \tfrac{1}{2}x.$$

The function $F_2(x)$ is known as Clausen's integral and has been tabulated[1] by him to 16D for the same values of x. The other functions F_k and f_k appear to be new. They are said to be useful in the theory of elasticity. Graphs of these functions are also given.

D. H. L.

[1] T. Clausen, *Jn. f. r. u. angew. Math.*, v. 8, 1832, p. 300, reprinted in F. W. Newman, *The Higher Trigonometry*, Cambridge, 1892, p. 85; see *MTAC*, v. 1, p. 458.

601[L].—Hermann Kober, *Dictionary of Conformal Representations.* Admiralty, Department of Physical Research, Mathematical and Statistical Section. Part V: *Higher Transcendental Functions.* v. 30, 3 leaves. Number SRE/ACS 111 = ACSIL/ADM/48/329. London, 1948. 20.1 × 33.1 cm. This publication is not available for general distribution.

This is the final part of the *Dictionary* of which the four earlier parts were reported *MTAC*, v. 2, p. 296–297; v. 3, p. 103. Elliptic functions are dealt with, leaves 1–24; and other functions, leaves 25–30. "References" are listed in the final 3 leaves.

602[L].—Wilhelm Magnus & Fritz Oberhettinger, *Formeln und Sätze für die speziellen Funktionen der mathematischen Physik.* (*Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete*, v. 52.) Second ed. rev. and enl., Berlin, Göttingen, Heidelberg, Springer, 1948, viii, 230 p. 16 × 24.3 cm.

In RMT 503 (v. 3, p. 103–105) we gave the detailed contents and list of errata of the first edition of this work, published in 1943. In the present edition the errata have been corrected and 58 pages added. Every chapter has been enlarged (although the fifth has the same number of pages) as may be observed by comparing the following inclusive chapter

page-numbers with those given earlier: I (1–9), Gamma function; II (10–24), Hypergeometric function; III (25–67), Cylinder functions; IV (68–101), Spherical functions; V (102–110) Orthogonal polynomials; VI (111–127), Confluent hypergeometric function and special cases; VII (128–158), Elliptic integrals, theta functions and elliptic functions; VIII (159–190), Integral transformations and inversions; IX (191–211), Coordinate transformations. Chapter VII has been entirely rewritten and doubled in size.

The first edition has been translated into English, and the New York publisher has informed me that he expects this translation to be available for distribution in January 1949.

R. C. A.

603[L].—SHEN YUAN, "The flow of a compressible fluid past quasielliptic cylinders at high subsonic speeds," Nat. Tsing Hua Univ., *Science Reports*, ser. A, *math. phys. Engin. Sci.*, v. 5, no. 1, Apr. 1948, p. 29–51.

On p. 44–45 are two tables of the Hypergeometric function $F(a, b, c; \tau)$ denoted for brevity as $F(m, \tau)$, where a, b, c, are functions of m as given by the equations $a + b = m - \frac{1}{2}$, $ab = -\frac{3}{4}m(m + 1)$, $c = m + 1$. T. 1: $m = [2(1)9(4)17; 4S]$, $\tau = .02(.02).16$; also $m = 30$, $\tau = .02, .04$. T. 2: $2m = [1(2)7; 4D]$, $\tau = .04(.02).3$; $2m = [41; 4S]$, $\tau = .04(.02).2$. Also $-2m = [1(2)15; 4D]$, $\tau = .04(.02)3$; also $-2m = [17; 4\text{–}6S]$, $\tau = .22(.02).3$; $-2m = [19; 4\text{–}6S]$, $\tau = .04(.02).3$; $-2m = [25; 4\text{–}5S]$, $\tau = .18(.02).3$; $-2m = [31; 1\text{–}4D]$, $\tau = .06(.02).3$; $-2m = [35; 4\text{–}6S]$, $\tau = .06(.02).22(.04).3$; $-2m = [41; 4\text{–}5S]$, $\tau = .06(.02).1, .18(.04).3$.

S. A. J. & R. C. A.

604[M].—ROBERT LECOLAZET & PHILIPPE PLUVINAGE, "États de régimes permanents electrodynamiques dans l'atmosphère," *Annales de Géophysique*, v. 4, 1948, p. 96–108. 21.5 × 27.4 cm.

On p. 102 there is a table of the exact values of $A_{m,n} = \int_0^1 P_m(u)P_n(u)du$, $P_n(u)$ a Legendre polynomial, for $m = 0(1)5$, $n = 0(1)5$.

605[M].—V. A. UGAROV, "K teorii strat" [Concerning the theory of strata], *Zhurnal eksperimental'noĭ i teoreticheskoĭ Fiziki*, v. 18, no. 5, May 1948, p. 457–461.

Table, p. 458, of $J(\Omega) = -4\int_0^\infty xe^{-x^2} \cos \Omega x dx$,

Ω	0	1	1.57	1.85	2	2.5	3	4	5	6	$10 \to \infty$
$J(\Omega)$	-2	-1.05	$-.32$	0	.16	.48	.56	.4	.2	.12	$.05 \to 4/\Omega^2$

EDITORIAL NOTE: Corresponding to $\Omega = 10$, the table gave the incorrect value of $J(\Omega)$ as .5.

606[U].—H. A. GULDHAMMER, *Nautisk Tabelsamling*. Copenhagen, J. Jørgensen & Co., 1946, 174 p. 16.8 × 25.5 cm.

This volume contains a collection of forty numbered tables and nine pages of mathematical formulae and unnumbered tables of equivalent weights, measures, etc. No explanation of the tables and their use is included. There is not even a preface nor an index, just a one-page table of contents. All table headings are in Danish; only one English phrase was noted in the volume—"Duration of rise or fall" in parentheses below the heading of T. 33. However a person who can read German and is moderately well acquainted with navigation, should have no difficulty in using the tables.

T. 1 gives the course or bearing in degrees and tenths corresponding to the points of the compass. T. 2 is one giving difference of latitude and departure, each to 0.1 for course angle 1°(1°)89° and distance 1(1)300. T. 3 provides the change in longitude to 0′.01 corresponding to departures 1(1)10 nautical miles [actually to 0′.001 for 10 nautical miles] and for middle

latitudes 1°(1°)60°(30′)70°(15′)80°. T. 4 is one of meridional parts to 0.1 for latitudes 0(1′)89°59′.

T. 5 gives the correction to 1′ to be applied to the middle latitude with arguments middle latitude 10°(10°)70° and change in latitude 1°(1°)15°. T. 6 yields the distance to the horizon to 0.1 nautical mile with argument height of eye 1(.5)10(1)30(2)100(5)170(10)200 meters. In T. 7 is tabulated the distance of an object to 0.1 nautical miles corresponding to height of the object 15(5)30(10)180, 200 meters and vertical angle subtended at the observer's eye 10′(5′)1°(10′)1°30′(15′)2°(30′)4°,5°. T. 8 gives the distance to 0.1 units of an object by two bearings with respect to the ship's course, with arguments first bearing 20°(5°)105° and second bearing 40°(5°)140°, the unit of distance being the distance run between the two bearings. There is also a brief section of this table with arguments given in points of the compass.

T. 9 gives the dip of the horizon to 0′.1 with argument height of eye .5(.5)18(1)42(2)66 meters. T. 10 gives corrections to 0′.1 to be applied to the dip of the horizon when the air is 1°(1°)10° Centigrade warmer or colder than the water. T. 11 provides corrections to 0′.1 to be applied to an observed altitude measured from a shore short of the horizon, the arguments being distance to the shore .5(.25)3(.5)5(1)7 nautical miles and height of eye 2(1)10(2)16 meters.

T. 12 is one of mean refraction to 1″ for 15° Centigrade and 760 mm. of mercury for measured altitudes of stars 0° to 90°. T. 13 gives the combined correction to 0′.1 for atmospheric refraction, semidiameter and height of eye 0.2(.5)8(1)18, 20 meters to be applied to observed altitudes of the lower limb of the sun 6° to 90°. T. 14–15 give the usual additional corrections to be applied for variable semidiameter of the sun and for upper limb of the sun for 15 intervals during the year. T. 16 is similar to T. 13, except that it includes no correction for semidiameter and hence is to be used with observed altitudes of stars and planets. T. 17–18 give the corrections to 1″ to be applied to the mean refraction 2′(2′)12′ given by T. 12 when the temperature is $-35°(5°) + 35°$ Centigrade and the barometric pressure is 700(10)780 mm. T. 19 provides the correction to 0′.1 to the observed altitude of the moon's lower limb 3° to 90° for horizontal parallax 52′(1′)61′, and semidiameter and atmospheric refraction. T. 20–21 give the moon's diameter and semidiameter to 0′.1 corresponding to horizontal parallax 52′(1′)61′. T. 22 gives the day-numbers corresponding to 60 (or for leap years, 61) days in the year.

T. 23 provides the means of changing hours and minutes of time to decimal fractions to .01 of a day. T. 24 is for changing points and quarterpoints of a compass into degrees and tenths. T. 25–26 are for conversion of arc into time and conversely. T. 27 gives the amplitude to 0°.1 of celestial bodies which are rising or setting; the arguments are declination 0(30′)18°(15′)23°45′ and latitude 0(5°)20°(2°)50°(1°)66°, 66.5°.

T. 28 gives the change in altitude to 0″.01 during the last minute before and the first minute after culmination for latitudes 0(1°)46°(2°)70°, 80° and declination, same and opposite name, 0(1°)24°. T. 29 provides to 0.1 the square of the time interval from culmination in minutes, the argument being time $0(1^s)29^m59^s$. T. 30 provides the time interval from the meridian to the horizon in hours and minutes in latitudes 0(2°)10°(1°)66° and for declinations, same name and opposite, 0(1°)34°. T. 31 is intended for use with a radio direction finder; it provides the correction to a bearing due to the convergence of meridians for a change in longitude 1°(1°)14° and mid-latitude 4°(4°)20°(2°)60°(4°)72°.

T. 32 is a small table for interpolating the quantity required to change a solar time interval into a sidereal time interval. T. 33 is intended to be used in calculating the fractional height of tide at half hour intervals, knowing the duration of rise or fall $3\frac{1}{2}(\frac{1}{2})8$ hours. T. 34 provides a wind-scale; T. 35 is a distance-travelled table giving distance to 0.1 nautical miles for 1(1)60 minutes at 1(1)28 knots. T. 36 gives for $n = 1(1)240$ the values of n^2, n^3, $\sqrt{n}$ and $\sqrt[3]{n}$; the latter two are given to 4D. T. 37 is a 5D table of logarithms of numbers 10000(1)10999, 1100(1)9999, with P.P. T. 38 is a 3D table of the natural values of all six trigonometric functions for an angle 0(1°)90°. T. 39–40 provide 5D logarithms of trigono-

metric functions; T. 39 for sine, cosecant and cotangent for an angle 0(0′.1)2°12′ and T. 40 all six functions for an angle 0(1′)90°, Δ.

This volume has a neat blue cover with two gold lines around the edge of the front cover; it is well printed on a good grade of white paper. It seems likely that a person who uses the table frequently will grow to be very fond of the book.

Charles H. Smiley

Brown University

Editorial Note: On p. 79, heading, *for* Tab. 39, *read* Tab. 30.

MATHEMATICAL TABLES—ERRATA

In this issue references have been made to Errata in RMT **579** (Uhler), **580** (France), **584** (Kariakin), **596** (Jahnke & Emde), **605** (Ugarov), **606** (Guldhammer); **N96** (Pitiscus).

145.—G. F. Becker & C. E. Van Orstrand, *Hyperbolic Functions*, Fifth reprint, 1942. See *MTAC*, v. 2, p. 311 and v. 3, p. 200.

Using the NBSCL, *Tables of the Exponential Function e^x*, second edition, 1947, I find the following values:

tanh 0.174 = 0.17226 50005 13, cosh 0.911 = 1.44446 49997 49,
tanh 0.932 = 0.73152 49994 56, tanh 1.381 = 0.88117 49957 43,
tanh 1.986 = 0.96302 50028 60.

The roundings of these values to five decimals had been left in doubt in MTE 129. These results indicate the following three "errors in excess of 5 units in the next succeeding place of decimals" in *Hyperbolic Functions*:

page	u	function	*For*	*Read*
109	0.174	tanh u	.17226	.17227
124	0.932	tanh u	.73153	.73152
145	1.986	tanh u	.96302	.96303

E. G. H. Comfort

Illinois Institute of Technology

146.—R. A. Fisher, "On the 'probable error' of a coefficient of correlation deduced from a small sample," *Metron*, v. 1, no. 4, 1921, p. 3–32. On p. 26–27 is a table of $\tanh^{-1} x = \frac{1}{2}[\ln(1+x) - \ln(1-x)]$, $x = [0(.01)\text{-}.9(.001)1; 7D]$, δ^4.

On checking this table with a 9D table recently computed in this Laboratory we found only a single small error. In $\tanh^{-1}$ .918, *for* 1.576 159 6, *read* 1.576 159 5; our 9-place value is 1.5761 59504.

Theodore Singer

Computation Laboratory
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Editorial Note.—In this same paper of Fisher there is a table, p. 28, of $\tanh^{-1} \overline{(x-2/x)} = \frac{1}{2}\ln(x-1)$, for $x = [1(1)100; 7D]$. This table is not listed in FMR, *Index*, although Speidell's tables, 1622, of $\frac{1}{2}\ln x$ and of $\frac{1}{2}\ln(1/x)$, for $x = [1(1)1000; 6D]$, are noted.

147. M. Kraitchik, *Recherches sur la Théorie des Nombres*, v. 1, Paris, 1924.

On p. 216–265 of this volume there is a table of the indices of the first 25 primes, modulo N, for all primes $N < 10^4$. The following errata were noted by E. Jacobsthal:[1]

p. 256	$N = 8191$	$q =$	17	29	31	47	53	89
	for	ind $q =$	716	4129	6704	6589	2527	3918
	read	ind $q =$	2693	2152	491	376	550	1941
p. 260	$N = 9137$	$q =$	17	43	59	71	97	
	for	ind $q =$	3099	2373	8781	427	2454	
	read	ind $q =$	3098	2372	8780	428	2453	

Another error on p. 256 may be noted, *for* $N = 5387$, *read* 8387.

D. H. L.

[1] E. Jacobsthal, "Correction de quelques erreurs dans la table d'indices de M. Kraitchik," K. Norske Viden. Selskab, *Fordhandlinger*, Trondhjem, v. 19, 1946, p. 1–2.

148. William Oughtred (1574–1660), *Table of Ln x*. 1618.

Napier's *Mirifici Logarithmorum Canonis Descriptio* was published in 1614 and the first edition of the English translation by Edward Wright (1558?–1615) was published by his son Samuel Wright in 1616. In 1618 there appeared a second edition in which the main text was identical with the first, but a new title-page was supplied, and also a remarkable anonymous 16-page appendix by William Oughtred (1574–1660). This Appendix was completely reprinted by J. W. L. Glaisher in his very notable article "The earliest use of the radix method for calculating logarithms, with historical notices relating to the contributions of Oughtred and others to mathematical notation," *Quart. Jn. Math.*, v. 46, April 1915, p. 125–197.

The reprinted Table on p. 142 gives what is equivalent to $10^6 \ln x$, $x = [1(1)10(10)100(100)1000(1000)10000(10000)100000(100000)900000$; 6D]. Glaisher pointed out that there was a misprint of 6 for 8 in the third decimal of ln 3, but the third decimal of ln 50 should be 2, not 1 and the first decimal of ln 20000 should be 9, not 8. Apart from these major misprints for the 54 values of x there are 51 last figure errors of from 1 to 6 units: 3 of 6 units; 8 of 5; 8 of 4; 15 of 3; and 9 of 2. There is also a supplementary table giving $\ln x$ for 18 values $x = 1.1(.1)1.9$ and $1.01(.01)1.09$ each to 6D. These are correct except for 10 unit errors in last decimal places.

Such was the extent of the equivalent of the first table of $\ln x$, although there was at that time no thought of exponents or bases as they were later conceived. Those desiring to understand the exact setting of the table in the mathematical thought of the time will naturally turn to Glaisher's study.

R. C. A.

UNPUBLISHED MATHEMATICAL TABLES

75[K].—Univ. of California, Statistical Laboratory, Berkeley, *Tables of the Bivariate Normal Distribution.*

This Laboratory has just completed work on a table of the Bivariate Normal Distribution. The quantity tabled is

$$L(h,k,r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^{+\infty} \int_k^{+\infty} \exp\left\{ -\frac{x^2 - 2rxy + y^2}{2(1-r^2)} \right\} dxdy.$$

A table of this kind was originally computed and published by Karl Pearson in the volume he edited, *Tables for Statisticians and Biometricians*, part II. 1931, p. 78–137, lii–lxxix. In this publication the arguments h and k are tabled for 0(.1)2.6. The argument r is for $-1(.05) + 1$. Values are given to 6D.

During World War II the values of $L(h,k,r)$ became necessary in connection with certain work in the Statistical Laboratory under a contract with the Applied Mathematics Panel, N.D.R.C. However, for the most part, the values needed corresponded to h, and k exceeding the range of the table then available. Also, the requisite values of r were extremely close to ± 1 in which region interpolation in Pearson's table is difficult.

The persistent need of values of $L(h,k,r)$ suggested that it might be useful to solve the difficulty once and for all by computing an extension of Pearson's tables. Dr. LEO AROIAN and Dr. MADELINE JOHNSEN (then in the employ of the Statistical Laboratory) were entrusted with calculating the extension of the tables. Their work was interrupted by the cessation of hostilities in September, 1945, and the subsequent discontinuance of the N.D.R.C. project. Thereafter the work on the tables continued sporadically. Some computations were done by Drs. Aroian and Johnsen. Later on a check of the tables was made in the Statistical Laboratory by Dr. EVELYN FIX. This was followed by extensive recalculations by Miss MARY WOO and Miss ESTHER SEIDEN under the direction of Dr. Fix. It is now believed that the errors in the table do not exceed one half unit in the sixth decimal. The combination of the two sets of tables, Karl Pearson's and the new tables, covers the ranges of $h,k = 0(.1)4$; $\pm r = 0(.05).95(.01).99$.

It may be hoped that some way will be found for the combined tables to appear in a single publication.

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AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. CANNON, 418 South Building, National Bureau of Standards, Washington 25, D. C.

TECHNICAL DEVELOPMENTS

Our contribution under this heading, appearing earlier in this issue, is "Coding of a Laplace boundary value problem for the UNIVAC," by FRANCES E. SNYDER & H. M. LIVINGSTON.

DISCUSSIONS

A Comparison of Various Computing Machines Used in the Reduction of Doppler Observations

Introduction. DOVAP (Doppler Velocity and Position) is a radio-DOPPLER method for the determination of the coordinates of the trajectories of long-range V-2 rockets launched at the Army's White Sands Proving Ground, New Mexico. The method has been in use for the past two years. In this system, continuous-wave radio signals are sent from a transmitter to a transceiver in the missile and to each of several ground station receivers. In the missile-transceiver the signals are modified by a frequency-doubling operation and then are retransmitted to each of the several ground stations. In the ground receivers the frequency of the signals received directly from the transmitter is likewise doubled, and these double-frequency signals are mixed with those received from the missile. The mixed signals are recorded on 35 mm movie film simultaneously for the several receivers at one master receiver station.

The film records present the traces of sinuous waves of changing frequency accompanied by a series of time pulses. Each successive cycle of the recorded wave indicates that the total distance from transmitter to missile to receiver has changed by one Doppler wave-length. If the initial (launching) value of this combined distance is known, all subsequent values are found simply by counting the Doppler cycles on the film record. Refraction and other physical effects will introduce differential errors, which are, however, ignored in the first approximations which are discussed here.

For V-2 missiles more than 50,000 cycles may have to be counted for the maximum ordinate of the trajectory. The distances from transmitter to missile to receiver involve numbers of six-digit accuracy. In the determination of one point on the trajectory, approximately 40 additions, multiplications, divisions and square roots are performed. The numerical work necessary for the calculation of the positions of the required number of points on a long trajectory justifies the utilization of high-speed computing machines.

The General Problem. In the determination of the missile location from DOVAP data, it is assumed that the coordinates of one point on the trajectory are known accurately. This point is usually the position of the transceiver in the missile at the time the missile is launched. If u_i is the distance from the transmitter to the missile to receiver i, the survey data of the launching and station sites furnish the initial values of $(u_i)_0 = c_i$. The difference between u_i and c_i at any time t can be obtained from the Doppler records. Given c_i and λ, the Doppler wave-length corresponding to twice the transmitted frequency, we have therefore, $u_i = c_i + N_i\lambda$, where N_i represents the number of Doppler cycles observed on the DOVAP records between the initial point of the ith trace and the point corresponding to the time t.

This paper treats the case in which three receivers are used in the DOVAP system. To determine the missile coordinates, three receivers are sufficient. The data from any one receiver prescribe that the missile is somewhere on the surface of a prolate spheroid, one of whose foci is located at the transmitter and the other at the receiver. The actual location of the missile is a common intersection of three such prolate spheroids having in common one focus, located at the transmitter. For three receivers there are two such solutions, one of which, generally being underground, is obviously rejected.

Briefly, the equations for the solution of the problem are:[1]

$$\sqrt{x^2 + y^2 + z^2} + \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} = u_i (i = 1,2,3),$$

x,y,z referring to the coordinates of the missile, and x_i, y_i, z_i referring to those of the receiver stations, relative to the transmitter as origin. If r and r_i are the corresponding slant ranges, the equations become,

$$2(x_i x + y_i y + z_i z) = 2ru_i - u_i^2 + r_i^2, \qquad (i = 1,2,3).$$

By substituting $2r = u_3 - \epsilon$, the solution of the problem is obtained rapidly by successive approximations in the following formulae, in the first approximation of which a value is assumed for z: $\epsilon = az_{(1)} + b$, $r_{(1)} = \frac{1}{2}(u_3 - \epsilon)$, $x_{(1)} = c\epsilon + d$, $y_{(1)} = e\epsilon + f$, $z_{(2)} = (r^2 - x^2 - y^2)^{\frac{1}{2}}$.

The corrections for successive approximations become: $\Delta z = z_{(2)} - z_{(1)}$, $\delta\epsilon = a\Delta z$, $\delta r = \frac{1}{2}\epsilon$, $\delta x = c\epsilon$, $\delta y = e\epsilon$, and $z_{(3)} = (r_{(2)}{}^2 - x_{(2)}{}^2 - y_{(2)}{}^2)^{\frac{1}{2}}$.

The coefficients a through f are all functions of u_i. More than two approximations in z are seldom necessary in the cases where the transmitter is close to one of the receivers. When, however, all receivers are located from 10 to 15 miles from the transmitter, as many as 5 approximations have occasionally been found necessary for the early parts of the trajectory in order to get agreement to within one foot in successive values for z.

Equipment Available for Computations. The computations of coordinates by human computers using desk machines (i.e., Friden, Marchant or Monroe) require between 15 and 45 minutes per trajectory point, the actual time required depending on the skill of the individual, his familiarity with the detailed formulae, and the number of approximations necessary. For detailed trajectory information, many points on the trajectory must be considered, and the computations by hand are laborious. For example, to compute the coordinates of the 800 points at half-second intervals on the trajectory of a 400-second time-of-flight missile, about 10 man-weeks would be required. Fortunately, this time can be greatly reduced by the use of modern high-speed computing machinery. In chronological order of availability, we at the BRL, Aberdeen Proving Ground, Md., have experimented with the following machines: 1. standard IBM equipment, 2. IBM Relay Multipliers, 3. the ENIAC, 4. the Bell Telephone Laboratories' Computing Machine.

The standard IBM equipment is impractical for the complete solution of the problem for several reasons. The large number of digits involved taxes the capacity of the multipliers. Furthermore, the problem has to be broken down into too many discrete steps, since the most complicated single operation that can be performed on these IBM machines[2] is of the form $A + Bx$. Moreover the capacity of a single card is soon exhausted, necessitating frequent reproduction of partial results onto new card sets in order that the computations may be continued. The multiplier operations are all slow, and machine break-downs are a frequent cause of interruption. Thus, while the complete computations for an 800-point trajectory should theoretically require much less than a week on this equipment, the actual time is more nearly comparable with human computer time.

The new relay calculators, which performed their initial work on this problem, are much more practical, though not nearly so expeditious as their theoretical operation rates would indicate. It has been our experience that about two weeks are required for the determination on a 400-second trajectory of the coordinates of positions, with successive first and second differences, and velocities at half-second intervals. (The theoretical time would be about 4 days.)

The Ballistic Research Laboratories have two of the new IBM relay calculators. These two units have been connected so that they may now operate as a single unit. Test runs indicate that this Siamese-twin computer will be able to determine missile coordinates at a rate of 5 to 8 minutes per point. Computations for sizeable runs of points have not as yet been completed by this new method. Hence, we can at present comment only on the promise of this arrangement; we cannot examine, by use of actual statistics, its long-range dependability.

The ENIAC is very much faster and more economical in the use of cards than the IBM relay calculators. Only the input data and required

results are punched on the cards. It is partly for this reason that the ENIAC can achieve a greater speed, since even the punching of results on the cards takes longer than the actual computing. The most time-consuming factor occurring in the use of the ENIAC is the "programming" (i.e. setting up the machine for a specific problem). For the particular DOVAP problem, the programming for the computations of only trajectory point coordinates requires from one to two days. Approximately one more day of programming would be required to determine the velocities in addition to the coordinates.

Machine failures are frequent on the ENIAC. 10% failure (in percentage of operating time) is not unusual; on a few problems the failure ratio has been as high as 75%. Nevertheless, the ENIAC excels all other present available machines in the speed with which it is capable of computing DOVAP results. Barring machine failures (including failures due to warped cards or cards affected by humidity or static charges) the actual computations of trajectories on the ENIAC require little more time than the time of flight of the missile. A computation time of 15 minutes for the 800-point trajectory would be normal. Clearly, even though the programming time for the ENIAC is relatively long, the machine is very efficient for the performance of a long series of calculations of the same type.

For the reduction of a limited amount of data, or for computing check points on very long ENIAC runs, the Bell Telephone Laboratories' Computing Machines (designed by Dr. G. R. STIBITZ) are more practical, though slower in performing the basic arithmetic operations than the other machines tried. These machines require approximately 5 minutes for the computation of the coordinates of a trajectory point, but the preparatory time for programming the problem prior to actual computations is negligible, once the data have been coded on the machine data tapes. These machines are now being used to reduce at half-second intervals the data pertaining to a missile flight through burn-out only (which requires about one minute of flight), and for points at 10-second intervals for all long flights. Although the time, 5 minutes per point, is comparable with the over-all time required by the IBM relay calculators, the efficiency of the Bell machines is greater. Thus far, these machines have made few errors. Only one error was found in the results for approximately 1200 points computed.

These machines have the added advantage over the IBM equipment in that they require less supervision. Provided that sufficient data tapes are stored in the Bell machines, they can be left operating all night. Moreover, in case errors in data or other handicaps which make a problem insolvable are encountered, the problem is automatically switched off, or the machines proceed to the next computation that the instruction tapes specify. The IBM machines do not have this facility. Because they are able to operate unattended all night, the Bell machines, can produce much greater output per operator hour than the IBM relay machines and indeed they run a close second to the ENIAC in utility on the DOVAP problem—the 800-point trajectory, requiring less than 70 hours, would take only three working days if run continuously.

To conclude, it has been found that the ENIAC is most efficiently used in the solutions of many complete runs of the same type of problem. Since the limiting factor in the use of this machine is the actual time required for

the programming of the problem, once the problem has been set up the ENIAC can provide the complete solution of many long DOVAP trajectory problems at amazingly high speed. The IBM machines, because of the relatively short set-up time and long computing time, are best suited to the simultaneous solution of one particular phase of the problem for all the trajectory points. In addition, very detailed checks are necessary, when the IBM machines are used, in order to determine the adequacy of these partial results before the completion of the total trajectory computation. Otherwise an error in any one of the 40 odd steps in the computations for one point might not be detected until the work for all 800 points had been completed. The Bell machines, however, are both flexible and reliable. They compute complete data for successive trajectory points, and they require very little set-up time. They are particularly useful for the solution of small, varied problems and are consequently desirable for providing the ENIAC with isolated check results whenever the data at hand are justifiably abundant for the utilization of the ENIAC.

It appears from the tests to date at the BRL that the complete DOVAP computations for a single 800-point trajectory are done equally expeditiously on the ENIAC or the Bell machines (and perhaps as efficiently on the twin IBM relay calculators—although limited experience with these machines precludes any accurate appraisal at the present time of their use in the solution of the DOVAP problem). Because of the high programming time of the ENIAC, however, the Bell machines are to be preferred for shorter problems like the computation for a few points on a single trajectory. For longer problems the picture changes. For example, the computations on 10 trajectories for which the raw data are available simultaneously would be completed in about 2½ days on the ENIAC, unless grave machine failures occurred (2 days preparatory plus 15 minutes per trajectory), but would require almost 30 days of continuous day and night running time on the Bell machines. Both the ENIAC and the Bell Relay Machine are extremely useful in the fast reduction of DOVAP data. By comparison, the standard IBM machines are quite limited in utility on this type of problem.

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EDITORIAL NOTE: Dr. Hoffleit's paper is a revision of the one to which we referred, *MTAC*, v. 3, p. 133 (7).

[1] The detailed mathematical solutions were carried out by Dr. BORIS GARFINKEL of the Ballistic Research Laboratories (BRL), Aberdeen, Md.

[2] We refer in particular to Multiplier Model 601. Newer models are both faster and more flexible.

BIBLIOGRAPHY Z–VI

1. HOWARD H. AIKEN & GRACE M. HOPPER, "The Automatic Sequence Controlled Calculator," II, III *Electrical Engineering*, v. 65, Oct.–Nov., 1946, p. 449–454, 522–528. 21.6 × 27.9 cm. See *MTAC*, v. 2, p. 185–187, p. 316; v. 3, p. 210.

These articles continue a discussion of the functions of the ASCC developed for Harvard University by the IBM Corporation. The calculator will carry out any selected sequence

of the four fundamental operations of arithmetic (addition, subtraction, multiplication, division), and reference to tables of previously computed results under completely automatic control. The functions considered herein are those of the multiplication and division registers and of the functional units. The preparation and planning of the sequence controlled tapes are discussed.

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2. ANON., "Relay computer for the army," *Bell Laboratories' Record*, v. 26, 1948, p. 208–209. 17.8 × 25.4 cm.

3. ANON., "The Univac," *Electronic Industries*, v. 2, May, 1948, p. 9, 19; illustr. 20.3 × 27.9 cm.

"An extremely high-speed electronic digital computer, designed by the Eckert-Mauchly Computer Corporation, Philadelphia, is an advance in the new science of planning industrial and military operations by solutions of formulas."

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4. JULIAN H. BIGELOW, JAMES H. POMERENE, RALPH J. SLUTZ, & WILLIS H. WARE, *Interim Progress Report on the Physical Realization of an Electronic Computing Instrument*, Princeton, N. J., Institute for Advanced Study, 1 Jan., 1947, viii, 101 leaves. Mimeographed. 21.6 × 27.9 cm.

This report had been previously circulated to some degree in preliminary form but has only recently been reproduced in quantity and made available for wider distribution. The contents appear to be identical with those of the copies previously issued.

The report describes the engineering phases of the work on electronic digital computers at the Institute for Advanced Study in an attempt to obtain the physical realization of the machine which was discussed from the logical standpoint in a companion report issued by the Institute, 28 June, 1946, entitled *Preliminary Discussion of the Logical Design of an Electronic Computing Instrument;* see *MTAC*, v. 3, p. 50–53 for a review of this report. Beginning with an outline of the objective and organization planned to meet it, the report continues with a general discussion of the arrangement and components of the entire computer. It then goes on to present a detailed description of analysis tests and conclusions drawn from them on the use of a magnetic wire or tape for external memory and operation of elementary circuit components such as the "flip-flop" and the "counter." The report ends with a brief statement of the future plans of the group at the time the report was issued.

It is to be noted that two years have elapsed since the date of this report. In a field as rapidly developing as electronic digital computing, this is a relatively enormous lapse of time. In addition, development of the "selectron" has been disappointing, at least, as concerns the time schedule. In view of these two facts, the report is somewhat outdated. It is believed, however, that much of the content is applicable even at this date, provided the reader overlooks the multitude of typographical errors scattered throughout the report. This is the more true in view of the relatively small amount of development that has taken place elsewhere in regard to input and output equipment for high-speed computers as compared to work on the more glamorous phases of high-speed memory, control, and electronic computation.

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5. LOUIS COUFFIGNAL, "Les travaux français sur les machines mathématiques," *Atomes*, v. 3, no. 23, Feb., 1948, p. 44–48. 21 × 26.7 cm.

6. Andrew V. Haeff, "A memory tube," *Electronics*, v. 20, 1947, p. 80–83. 20.3 × 29.8 cm. See *MTAC*, v. 3, p. 281–286.

A pattern is produced, stored, and scanned on dielectric screen by electron beams. The special cathode-ray tube can be used to store for protracted periods television pictures, radar indicator patterns, oscilloscope traces, or other information.

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7. Andre Lion, "Atom taming equations turn to peace tasks—pure science, higher mathematics get bigger industrial roles," *Barron's National Business Weekly*, Jan. 6, 1947. Newspaper.

8. Moore School of Electrical Engineering, University of Pennsylvania, *Progress Report on the EDVAC* (Electronic Discrete Variable Computer). Prepared by the staff of the EDVAC project and edited by T. Kite Sharpless. 2 v., Philadelphia, Pa., June 30, 1946, about 600 leaves, tables and diags. 21.6 × 27.9 cm.

The Ordnance Department of the United States Army, seeing the need for tremendous amounts of computation in military and civilian scientific work, is sponsoring the design and construction of an extremely fast electronic computer at the Moore School of Electrical Engineering of the University of Pennsylvania.

The EDVAC Report presents the state of thinking about electronic computers at the Moore School at the date of writing (1946). The content of the Report is largely determined by one of its objectives, namely, to give the Ordnance patent department material on which to base patent applications. For this reason, and because it was written while the project was in the early exploratory stage, the Report describes a great many conceivable ways of designing parts of the computer. For example, a chapter on "adders" treats a number of circuits using standard triodes, standard pentodes, and tubes associated with resistance matrices or "function tables," as well as proposed special adder tubes; and these elements are combined into adders for binary, bi-quinary, shifted binary, and decimal notations.

The report contains nine chapters dealing with the components of a digital computer and their organization. Chapter I deals with adders, multipliers, miscellaneous circuits, and "computers." The discussion on adders has been mentioned, and the remarks about that section apply to the sections on other components. Among the miscellaneous circuits are gating circuits to control the flow of signals along a path, circuits to produce complements of numbers, and circuits that will in effect align the "binary point" when binary numbers of different orders of magnitude are to be added. The computer section explains how adders and multipliers can be controlled to carry out the fundamental arithmetic operations.

Chapter II is a general discussion of acoustic wave propagation in tubular spaces, with special attention to the generation, propagation, detection, and distortion of signal pulses thru tubes containing piezoelectric crystal transducers for converting electrical into acoustic pulses. The chapter includes a detailed theoretical analysis of reflection at the ends of the tube, the cutting and mounting of the crystals, and the characteristic impedances of the resulting acoustic delay line. Data in the form of a sequence of pulses can be fed into one end of a delay line, picked up at the other end, reshaped by an amplifier and fed back into the line. Fairly complete circuits for the reshaping amplifier are given, and the problem of synchronizing several delay lines, so that they can work in parallel, is treated.

Chaper III is a short discussion of ways of remembering data in cathode-ray tubes (similar to television tubes) and in the RCA Selectron. Chapter IV describes briefly, and in very general terms, magnetic recording on tapes and discs, and a calculator based on such recording. Chapter V points out the requirements of the typewriter, or manually operated input device, and of the printer, or automatically operated output, for a computer. Several proposals for each are mentioned. In a large computer, the equipment needed to steer

numbers and instructions over the various possible paths within the computer is a major part of the entire machine. Chapter VI treats this part of the subject in some detail. The last three chapters of the report deal with various proposed forms of EDVAC, namely, a Serial Acoustic, a 4-Channel Electrostatic, and a 4-Channel form.

The EDVAC Report suffers less from incoherence than most group reports; it is carefully arranged and edited, and records a wide variety of ideas in accessible form. In many ways it is unfortunate that publication of reports on computer techniques must be so long delayed. A review of a report from the EDVAC group on the changes in the picture that have occurred in the two and one half years since the present report was written would be valuable.

GEORGE R. STIBITZ

9. Institute of Radio Engin., *Proc.*, v. 36, Mar., 1948, p. 377. 20.3 × 27.9 cm.

On this page are abstracts of papers dealing with high-speed computers, presented at a meeting of the Institute, March 24, 1948.

MDL

NEWS

Association for Computing Machinery.—The proposed form, dated February 15, 1948, of the Constitution and Bylaws for the Association, sent out to the members for balloting, was not adopted. Because of the high proportion of ballots in favor of its adoption, however, the Council of the Association, at a meeting on May 27th, resolved "to act in accordance with the proposed Constitution and Bylaws." The Association's Committee on Constitution and Bylaws will consider the suggestions for its revision (particularly the proposed limitation of the number of Members-at-Large on the Council, and the proposed specification of the manner in which election ballots shall be counted), formulate their recommendations, and submit them to the Council. An improved draft of a proposed Constitution and Bylaws will then be resubmitted to the members of the Association. Also at its May 27th meeting, the Council resolved to hold elections promptly for President, Vice-President, Section Officers, and Members-at-Large, for the period until May 31, 1949. The President, John Curtiss, appointed a nominating committee consisting of G. R. STIBITZ, S. N. ALEXANDER, and C. V. L. SMITH. They met on June 10, 1948, and made the following nominations: for president: J. W. MAUCHLY (E-MCC); for vice-president: F. L. ALT (BRL, Aberdeen); for section officer—s.o. (Boston): F. L. VERZUH (MIT); for s.o. (New York): SAMUEL LUBKIN (NBS); for s.o. (Philadelphia and Aberdeen): T. K. SHARPLESS (Technitrol Engin. Co.); for s.o. (Washington): MINA REES (ONR); for member at large—m.a.l. (mathematics): HANS RADEMACHER (Univ. Pa.); for m.a.l. (statistics): J. L. MCPHERSON (BC); for m.a.l. (communications): C. B. TOMPKINS (Engin. Res. Assoc.); for m.a.l. (business and finance): HENRY RAHMEL (A. C. Nielsen Co.); for m.a.l. (engineering): CHARLES CONCORDIA (G.E. Co.). The Secretary and Treasurer are elected by the Council. The Association now has 465 members; a roster of members as of May 21, 1948, was prepared and distributed to the members.

The Institute for Teachers of Mathematics.—The 8th annual session of the Institute for Teachers of Mathematics was held on August 9–20, 1948, at Duke University, Durham, N. C. At that time 24 papers were read illustrating the use of mathematics in various scientific fields of endeavor, and many laboratory classes were held for the benefit of the participating teachers.

One of the sessions was given over to a discussion of automatic digital computing machines by Mrs. IDA RHODES of the NBSMDL. A brief history of computation and tools for computation preceded a detailed discussion of the electronic machines presently being constructed.

International Business Machines Corporation.—During the week of August 23, 1948, the IBM conducted a Scientific Computation Forum, which was attended by some 70 invited guests.

The first 4 days, sessions were held at Endicott, New York, and 20 papers were presented, dealing with basic techniques for IBM machine computation as applied to differencing of tables, matrix operations, differential equations, and numerous other branches of applied mathematics. A welcome feature of the meeting was the demonstration of the principles and uses of the soon-to-be-released IBM machine no. 604, which, because of its several new features, promises to be a most useful addition to any IBM installation engaged in computation for applied mathematics.

A tour through the IBM factory, guided by a member of the company, and a banquet held in the IBM Homestead completed the first stage of the forum. The last day was spent at the IBM World Headquarters in New York, where 4 additional papers were read and a demonstration of the IBM Selective Sequence Electronic Calculator was given.

The persons participating in this forum represented various government agencies, industrial organizations and academic institutions. Not only were the 24 papers informative and thought provoking, but the opportunity for exchange of opinion was of substantial benefit to anyone faced with computation problems.

Symposia on Modern Calculating Machinery and Numerical Methods.—The symposia were held July 29–31, 1948, at the University of California, Los Angeles, under the joint auspices of the Institute for Numerical Analysis (INA), NBS, and the Departments of Astronomy, Engineering, and Mathematics, UCLA, in cooperation with the AIEE, the Amer. Math. Soc., the Amer. Phys. Soc., the ASME, the ACM, the Engineering Division, Air Materiel Command, U.S.A.F., the Institute of the Aeronautical Sciences, the IRE, the Math. Assoc. Amer., and the ONR. In a sense, the symposia served as a continuation of the very significant symposium on large-scale digital calculating machinery which was held at the Harvard Computation Laboratory, Harvard University, Jan. 7–10, 1947; see *MTAC*, v. 2, p. 229–238.

The symposia provided those in attendance with up-to-date information regarding the technological and mathematical developments in the field of ultra-high speed numerical calculation. Also the symposia marked the formal opening of the INA which is a section of the NBSNAML. The establishment of the Institute was fostered by the Office of Naval Research (ONR) and has been firmly supported by the ONR and the Air Materiel Command of the USAF. The Institute functions as a center for basic research and training in the types of mathematics essential to the exploitation and the further development of high-speed automatic digital computing machinery; also it provides a computation service for the Southern California area and is concerned with the formulation and analytical solution of important problems in applied mathematics. In addition to the desk calculators and punch-card equipment already installed, the INA will be equipped with at least one general purpose large-scale electronic digital computing machine, as soon as such equipment becomes available.

Program

July 29, **Session I:** L. M. K. Boelter, chairman

Addresses of Welcome:

For the University: Clarence Dykstra, provost UCLA

For the NBS: W. R. Brode, assoc. director NBS

For the Navy Dept.: A. T. Waterman, ONR

For the USAF: O. C. Maier, Wright Field

Invited address: "Electronic methods of computation" by John von Neumann

Session II: J. H. Curtiss, chairman

"General survey of current British developments" by D. R. Hartree

"General survey of current American developments" by Perry Crawford, Jr.

Session III: *Progress Reports from Principal Academic Research Centers*, PAUL MORTON, chairman

"The electric analog computer" by G. D. MCCANN

"Comments on the reliability of operation of computing machinery" by H. H. AIKEN

"Project whirlwind at MIT" by J. W. FORRESTER

"Recent developments at project EDVAC" by R. L. SNYDER

"Recent developments at the Institute for Advanced Study" by H. H. GOLDSTINE

"Recent developments at the Illinois Inst. of Techn." by T. J. HIGGINS

July 30, **Session IV**: *Progress Reports from Principal Commercial Research Laboratories*, N. E. EDLEFSEN, chairman

"Recent developments—UNIVAC" by J. W. MAUCHLY

"Recent developments—REEVAC" by H. I. ZAGOR

"Recent developments—Engin. Res. Associates" by C. B. TOMPKINS

"Recent developments in electronic computers—IBM" by R. R. SEEBER

"Recent developments—Bell Tel. Labs." by B. MCMILLAN

"Recent developments—Raytheon Labs." by R. V. D. CAMPBELL

Session V: *Programming for Automatic Digital Computing Machinery*, JOHN TODD, chairman

"Programming for the Dahlgren machine" by C. C. BRAMBLE

"Programming for the Aberdeen machines" by FRANZ ALT

"Programming for the IBM SSEC" by R. R. SEEBER

"Programming for machines under development" by H. D. HUSKEY

"Programming for machines under construction" by IDA RHODES

Session VI: *Lecture* (illustrated with lantern slides), "Recent developments at the Harvard Computation Laboratory" by H. H. AIKEN. Also

General Open Discussion

H. H. AIKEN, *chairman*. F. L. ALT, J. W. FORRESTER and JOHN TODD assisted in answering questions.

July 31, **Session VII**: *The Future of Numerical Analysis*, E. F. BECKENBACH, chairman

"Some unsolved problems in numerical analysis" by D. R. HARTREE

"Numerical methods in pure mathematics" by D. H. LEHMER & HANS RADEMACHER

"Problems in probability and combinatorial analysis" by S. M. ULAM

Session VIII: *Numerical Methods in Applied Mathematics*, JOHN BARNES, chairman

"Numerical calculations in nonlinear mechanics" by SOLOMON LEFSCHETZ

"Programming in a linear structure" by G. B. DANTZIG

"Wave propagation in hydrodynamics and electrodynamics" by BERNARD FRIEDMAN

"Eigenvalues and eigenvectors for symmetric matrices" by H. H. GOLDSTINE

The following 515 members registered for the Symposia:

H. H. Aiken, Harvard Univ.
H. F. Allen, Head, Math. and Physics Dept., Coalinga Jr. College
F. L. Alt, BRL, Aberdeen Proving Ground, Md.
Alphonso Ambrosio, Engin. Dept., UCLA
B. F. Ambrosio, USN Electronics Lab., San Diego 52, Cal.
Ruth K. Anderson, USN Ordn. Test Station, China Lake, Cal.
E. J. Andrews, N. A. Aviation, Aerophysics Lab.
Selma Anno, Chicago, Ill.
R. F. Arenz, USNEL
W. N. Arnquist, ONR, Pasadena, Cal.
K. J. Arrow, Univ. Chicago
S. E. Asplund, AMS, Air Weather Service, USAF
H. T. Avery, Marchant Calculating Co., Oakland, Cal.
H. A. Babcock, Cons. Eng., USC
L. L. Bailin, NBS, UCLA
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W. W. Baldwin, Cons. Eng., Henry A. Babcock, Los Angeles, Cal.
Alfred Banos, Jr., Physics Dept., UCLA
J. L. Barnes, UCLA
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Richard Bellman, Stanford Univ.
W. W. Beman, Telecomputing Corp.
B. M. Bems, Shell Development Co.
E. T. Benediht, N. A. Aviation
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David Blackwell, Howard Univ., Washington, D. C.
Gertrude Blanch, NBSINA, UCLA
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C. Braudon, New Jersey
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Olga T. Todd, NBSINA, UCLA
L. A. Tolve, U. S. Air Force, AMC, Wright Field, Dayton, Ohio
C. B. Tompkins, Engin. Res. Assoc., Arlington, Va.
V. N. Tramontini, Engin. Res., UCLA
J. W. J. Truran, British Joint Services Mission
Mary J. Tudor, NBS
K. B. Tuttle, Northrop Aircraft Co.
A. W. Tyler, Eastman Kodak Co., Rochester, N. Y.
E. F. Tyler, Douglas Aircraft Co.
G. W. Tyler, USNEL
S. M. Ulam, Los Alamos Lab., and UCLA
F. A. Valentine, Math. Dept., UCLA
H. A. Van Dyke, USNOTS
C. J. Van Vliet, Dept. Meteor., UCLA
Andrew Vazsonyi, USNOTS
Ralph Vernon, Math. Instr., Claremont Men's College
B. L. Waddell, Northrop Aircraft Co., Hawthorne, Cal.
J. A. Widemnaun, N. A. Aviation
W. P. Wallace, Engin. Dept., UCLA
J. E. Walsh, Douglas Aircraft Corp., Santa Monica, Cal.
S. S. Walters, Math. Dept., UCLA & Hughes Aircraft Co.
L. E. Ward, AMS
Mary C. Ward, USNOTS
J. E. Warren, Cal. Res. Corp.
A. T. Waterman, ONR, Washington, D. C.
J. H. Wayland, USNOTS
Alex Wayman, Student, UCLA
J. H. Weaver, Telecomputing Corp., Burbank, Cal.
J. W. Webster, NEPA, Fairchild Engine & Aircraft Co.
Joseph Weinstein, Signal Corps Engin. Labs., N. J.
E. T. Welmers, Bell Aircraft Corp., Buffalo, N. Y.
R. L. Wenick, USC
R. L. Westhafer, New Mexico College of A. & M. A.
D. E. Whelan, Jr., U. S. Coast & Geodetic Survey
J. R. Whinnery, Hughes Aircraft Co.
W. B. White, Douglas Aircraft Corp., Santa Monica, Cal.
I. L. Wieselman, Northrop Aircraft Co., Hawthorne, Cal.
Weston Wilsing, Yakima Valley Jr. College
O. B. Wilson, Grad. Student, Physics Dept. UCLA
Edna B. Winter, Quincy, Ill.
H. A. Wood, Chance Vought Aircraft
W. L. Wood, U. S. Army Ordn. Dept., Computing Lab., BRL
W. W. Woodbury, Northrop Aircraft Co., Hawthorne, Cal.
B. M. Woods, Engin. Dept., UCLA
P. E. Wylie, UCLA
John Wyreen, UCLA
R. M. Yoder, USNAMTC
W. E. Young, Garden & Williams
E. C. Yowell, NBSINA, UCLA
S. T. Yuster, Petroleum Eng., Penn. State College
H. I. Zagor, Reeves Instrument Corp., N. Y.
H. A. Zartner, Dept. Meteor., UCLA
F. W. Zehan, Jet Prop. Lab., Cal. Tech.
G. A. Zizicas, Grad. Student, Engin. Dept., UCLA

OTHER AIDS TO COMPUTATION

Bibliography Z-VI

1. Kingsland Camp, "A duodecimal slide rule," *The Duodecimal Bull.*, v. 4, no. 2, Oct. 1948, p. 7–12. 14 × 21.6 cm.

A discussion of problems involved in producing such a computing instrument.

2. N. A. Carle, "Graphic presentation for solving cubic equations," *Civil Engineering*, New York, v. 18, June 1948, p. 53. 20.9 × 28.6 cm.

The equation $x^3 - 11.52x + 9.61 = 0$ is solved by means of three calculated tables, in connection with the simultaneous equations $y = x^3$, $y = 11.52x - 9.61$.

3. L. J. Comrie, "Regnekunst—i Fortid, Nutid og Fremtid," *Naturens Verden*, Copenhagen, v. 32, 1948, p. 140–156. 15.7 × 23.2 cm.

This is a Danish translation of the English article to which we have referred *MTAC*, v. 3, p. 53–54.

4. Ray E. Gilbert, "New assay slide rule computes complex ore values," *Engineering and Mining Jn.*, v. 149, no. 6, June 1948, p. 95–97. 21 × 28.6 cm.

Description of an assay slide rule now in use at the Mayflower mine of the New Park Mining Co., Park City, Utah, for determination of ore value, where factors taken into account include price of metal and grade of ore, and changes in smelter penalties and bonuses related to grade of ore.

5. Donald F. Othmer, Samuel Josefowitz, & A. E. Schmutzler, "Correlating densities of liquids. A new nomograph," *Industrial and Engineering Chemistry*, v. 40, May 1948, p. 883–885. 20.3 × 28.2 cm.

Summary: "Densities of liquids may be plotted directly to give straight lines on logarithmic paper against a temperature scale developed from the critical temperature and densities of a reference liquid. The method of correlation follows the technique used for vapor pressures, viscosities, surface tensions, and other physical properties of liquids and gases and is then expanded into a nomograph which allows direct determination of the density of organic liquids at any temperature when the density for that liquid at two different temperatures is known. Mathematical derivations indicate the theoretical basis of this plot and the resulting nomograph."

6. R. Hume Purdie, "The use of the Burroughs type 70 stock accounting machine for preparation of tables of values of polynomial functions," Soc. Chemical Industry, London, *Chemistry & Industry*, no. 17, Apr. 24, 1948, p. 265–266. 21.6 × 28 cm.

7. Raymond Redheffer, "A machine for playing the game of nim," *Amer. Math. Mo.*, v. 55, June–July 1948, p. 343–349.

The game of nim was named and first described in print by Professor C. L. Bouton in "Nim, a game with a complete mathematical theory," *Annals Math.*, s. 2, v. 3, p. 35–39, 1901. Then followed E. H. Moore, "A generalization of the game called nim," *Annals Math.*, s. 2, v. 11, p. 93–94, 1910.

"In 1940, E. U. Condon, director of the Nat. Bureau of Standards, obtained a patent on a machine for playing the normal case of the game forming the subject of this article.[1] The circuit appears to be quite different, however, from that considered here, and makes extensive use of relays. A model of this machine for four piles with a maximum of seven objects in each was actually built as an exhibit for the New York World's Fair. Since that machine contained over a ton of equipment, while the present one weighs only about five pounds, it is felt that this article will be of interest."

[1] E. U. Condon, G. L. Tawney, W. A. Derr, "A machine to play the game of nim," U. S. Patent no. 2,215544, Sept. 24, 1940.

8. H. Richardson, "Slide rule solves gas flow and dust sampling problems," *Chemical Engineering*, v. 55, no. 7, July 1948, p. 124–125. 20.5 × 28.6 cm.

Quotation: "During 14 years of test work in gases I have developed the special slide rule scales shown here for the solution of the more commonly used calculations in making velocity measurements with the pitot tube, in determining the vapor pressure and moisture

content of air from the dewpoint temperature, and in finding the proper rate of sampling in dust determinations."

9. PETER L. TEA, "A mechanical integraph for the numerical solution of integral equations," Franklin Institute, *Jn.*, v. 245, May 1948, p. 403–419.

Excellent mathematical discussion and integraph description, together with a table of results obtained by BUCKLEY and HEDEMAN, already referred to by us in reviewing an earlier paper by TEA (*MTAC*, v. 2, p. 41–42).

10. B. C. WILKAS, "Nomograph solves equations for laboratory soil permeability coefficient," *Civil Engineering*, New York, v. 18, June 1948, p. 51–52. 20.9 × 28.6 cm.

The equations in question are $K_T = .0738RFL_S'/t'H_{WC}$, and $K_{20} = K_T\nu_T/\nu_{20}$.

11. HEINZ WITTKE, *Die Rechenmaschine und ihre Rechentechnik. Eine Einführung und ein Übungsbuch mit ausgewählten Anwendungsbeispielen aus der Geodäsie, Geometrie und angewandten Mathematik* (*Sammlung Wichmann*, v. 12). Berlin-Grunewald, Herbert Wichmann, 1943, viii, 161 p. 17.1 × 24.9 cm.

NOTES

96. BARTHOLOMÄUS PITISCUS (1561–1613).—It is the purpose of this Note to summarize some information about Pitiscus and his mathematical work, and to give references to the sources where further details may be gleaned.[1–18] We shall particularly try to give comprehensive indications of his activity in connection with the publication of mathematical tables, and their editions. Here certain facts not mentioned in any of the sources below, and others rarely noted, shall be presented.

Very little is known concerning the life of Pitiscus who was born near Grünberg in Silesia. He pursued theological studies in Heidelberg and for more than a score of the last years of his life he was court chaplain and court preacher for Elector FREDERICK IV of the Palatinate. During these latter years he published various editions of a Trigonometry, and Mathematical Tables, and edited and published, just before his death in 1613, the fine sine tables of RHETICUS (1514–1576).

The word Trigonometry is due to Pitiscus[18] and was first printed in his

1. *Trigonometria: sive De Solvtione Triangvlorvm Tractatus breuis & perspicuus*, 57 p. which was published as the final part (p. 157–213) of the following work by ABRAHAM SCULTETUS[7] (1566–1625) Professor of theology at the University of Heidelberg: *Sphaericorvm Libri Tres Methodicè conscripti & utilibus scholiis expositi*. Heidelberg, 1595, 213 p. This Pitiscus *Tractatus* was developed into the [viii] 371-page volume (2 uncounted white p. between p. [214] and 215),

2. *Trigonometriæ siue De dimensione Triangulor Libri Qvinqve. Item Problematvm variorū. nempe Geodaeticorum, Altimetricorum, Geographicorum, Gnomonicorum, et Astronomicorum: Libri Decem Trigonometriae Svbivncti, Ad Vsvm Eivs Demon-Strandvm*. Augsburg, 1600. The Trigonometry ends

on p. 122 and the special title-page, 123 (without place or date of publication),

3. *Canon Triangulorum Siue Tabulæ Si-Nvvm, Tangentivm Et Secantivm Ad partes radij 100000. & ad scrupula prima Quadrantis*, is followed by the table p. 124–213. The remaining pages (215–370) are occupied with Problemata Varia, this section having its own dated title page. The date CIↃ IↃ IↃ seems to have been interpreted as a misprint for CIↃ IↃ IC = 1599, which agrees with the date given in the astronomical bibliographies of J. F. Weidler, and J. de Lalande (copied from Weidler). But since the date is here incorrect one may just as well argue that for the second IↃ, C was intended, making the date 1600 for p. 215–370, which is much more reasonable following p. 1–214 of the *Trigonometria* and *Canon*, after a title-page dated 1600, and especially since pagination and signatures are continuous, and since in the colophon on p. [371], part of a signature, we find: "Avgvstae Vindelicorvm, | typis Michaëlis Mangeri, | Sumptibus Dominici Custodis Chalchograqhi. | M.DC." Hence the only date properly used in connection with this trigonometry is 1600. Of this work we used the University of Michigan copy which lacks the 4 p. (1 for errata and 3 white) inserted between p. 370 and [371] in the copy used by Gravelaar.[7]

In the brief gnomonic part of the *Problemata*, spherical trigonometry is employed in astronomical problems. No single word is written about the motion of the earth which for a theologian of those days was doubtless prudent silence.

4. The second enlarged edition of **2**, containing viii, 334, 219, 3 p. errata mostly in the *Trigonometria*, was published at Augsburg in 1609 (p. 334), not 1608, although 1608 is the date on the title-page. The trigonometry now occupies p. 1–172 and is followed by the Problemata Varia, p. 173–333.

5. The largely expanded tables, *Canon Triangvlorvm Emendatissimvs* (219 p.), are separately paged (at the end of the volume) and have their own title-page, dated 1608.

6. Still another arrangement occurs in the third edition of **2**, appearing at Frankfort in 1612. There are 3 sections, each with its own title page. The first section viii, 183, 2 p. is the *Trigonometria*, and the third section, separately paged, 270 p., is devoted to the Problemata Varia.

7. The second section is the *Canon*, unpaged [219 p.] and with a quite different type-display from **5**, and is followed by 3 p. of errata in the *Canon*.

In the dedicatory epistle to Elector Frederick IV, in nos. **2**, **4**, **6**, the following passage occurs: "Good God! How great and how rare an ornament is affability among theologians! And how thoroughly desirable would it be in this age that all theologians be mathematicians, that is, that they be reasonable and gentle men."

We shall here pause to describe the contents of the tables, which all give the natural values of all six of the trigonometric functions. No. **3**, containing [91] p., is a 5–6D table at interval 1′.

In nos. **5** and **7** the intervals are 0(1″)1′(2″)10′(10″)1°(1′)45°, with PP 10″. It is a 7D table for sin, cos; 7–8D table for tan, cot; 8–9D table for sec, csc.

Many writers have declared that Pitiscus used the decimal point in his Trigonometry and Tables but Cajori has shown[11] that such writers are mistaken; Pitiscus did not use the decimal point.

8. There were three English editions[15–18] of the Trigonometry, alone, of Pitiscus: (**a**) 1614, [xi], 176, 33, [2] p.; (**b**) 1630 [x], 210 p.; and (**c**) [1631], [viii], 208 p. This translation was made from no. **6**, p. 1–183. The title of (**b**) is as follows: *Trigonometrie | or | The Doctrine Of | Triangles. | First written in Latine, by | Bartholomevv Pitiscvs | of Grunberg in Silesia, and now | Translated into English, | By Ra: Handson. | Whereunto is added (for the Marri- | ners vse) certaine Nauticall Questions, toge- | ther with the finding of the Variation of | the Compasse. All performed Arith- | metically, without Map, Sphære, | Globe, or Astrolabe, | by the said | R. H.* [London] Printed by B. A. and T. | Fawcit for J. Tap. [1630]. The *Trigonometrie* occupies p. 1–176. On p. 210 is London. | Printed by B. Alsop and T. Favvcit for Iohn Tap, and | are to be sold at his shop at St. Magnus Corner. | 1630. | It will now be desirable to indicate also the title-page for **8**(**c**) given by SAMPSON[10]: *Trigonometry: | Or, The | Doctrine | Of | Triangles. | First written in Latine, by | Bartholomew Pitiscvs | of Grunberg in Silesia, and now | Translated into English, | by Ra: Handson. | Whereunto is added (for the Mariners | use) certaine Nauticall Questions, to- | gether with the finding of the Variation of | the Compasse. All performed Arith- | metically, without Map, Sphære, Globe, or Astrolabe, | by the said R. H. | Printed by T. P.*[*urfoot*] *for G. Hurlock | neare Magnus Corner* | s. l., s. a. size $5\frac{1}{2} \times 7\frac{1}{4}$ inches.

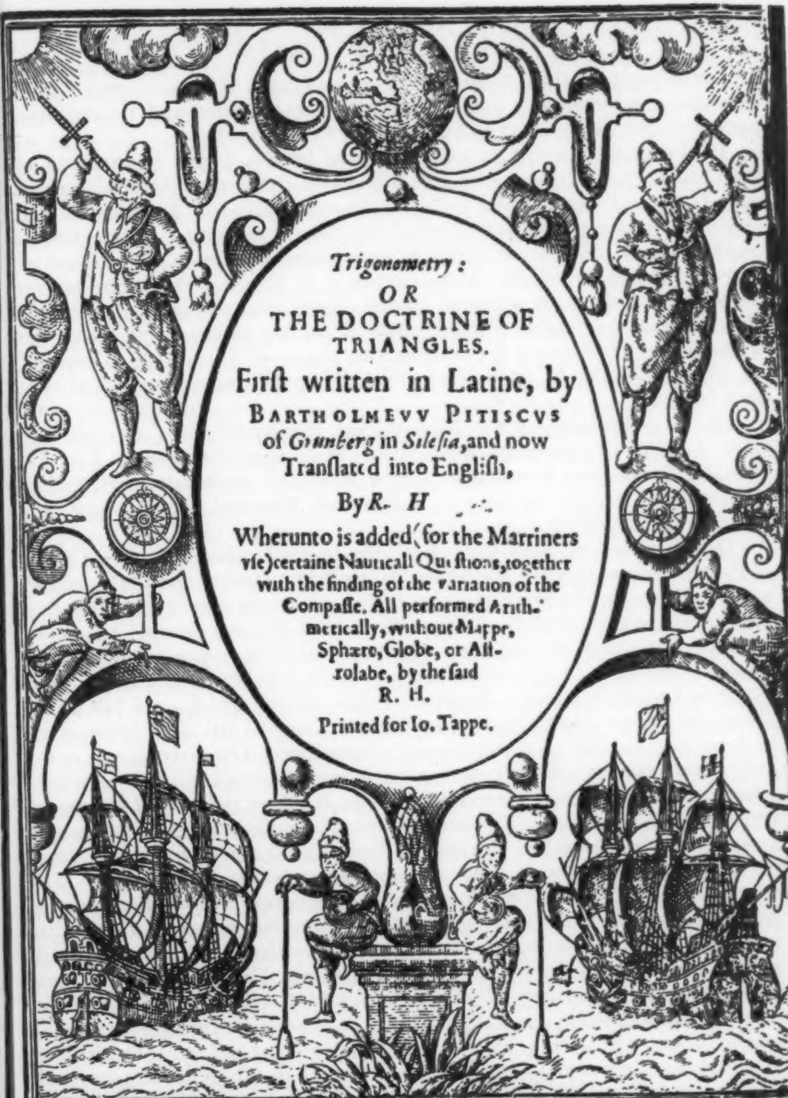
The differences of the title-pages of **8**(**b**) and **8**(**c**) are thus evident. It is therefore clear not only that SAMPSON was incorrect in stating that the undated **8**(**c**) of the Crawford Library was the 1630 edition, but also that the Crawford Library *Catalogue*[12] may now have [1631] added to its entry, and that owners of *STC* should list the Crawford Library under 19968a.

Through the kindness of Mr. HORBLIT, who bought a copy of **8**(**a**) in 1948, we were allowed to reproduce its title page, in which the name of the translator has been almost wholly erased. From the Huntington Library we learn that the original of this mutilated line was

By Ra: Handson.

Colophon: LONDON. | Printed by *Edw: Allde* for *Iohn Tap*, and are to be sold at his | shop at *St. Magnus* corner. | 1614. |

9. With **8**(**a**) and (**b**) were published English editions of **3**: **9**(**a**) *A Canon of Triangles: or The Tables, of Sines, Tangents & Secants, the Radius assumed to be 100000.* [91] p. In the 1614 edition there was nothing else on the title-page. But in the second English edition, **9**(**b**), entirely reset, one finds "London, Printed for Iohn Tap. 1630." In the case of known copies of **8**(**c**), **9**(**b**) is bound in. The Brown Univ. copy of **9**(**a**) is untrimmed and of size 14.3 × 18.9 cm. No. **9**(**a**) is in this Note identified in print for the first time, although the *STC* gave the date, 17 Jan. 1614, of its entry in the Stationers' Companys' Register. Furthermore, that **9**(**a**), (**b**) are English editions, with corrections, of **3** is here indicated for the first time. To illustrate corrections: the value of tan 1°44′ is given in **3** as .03926 which is entered correctly in **9**(**a**) as .03026; in the column of values of tangents under 8°, the errors in **3** of 5 to 10 units in the fifth decimal place for 33′, 36′ and 55′ are similarly corrected. The *form* of title-page for **9**(**a**) is practically



Trigonometry:
OR
THE DOCTRINE OF
TRIANGLES.
Firſt written in Latine, by
BARTHOLMEVV PITISCVS
of *Grunberg* in *Sileſia*, and now
Tranſlated into Engliſh,
By *R. H*
Wherunto is added (for the Marriners
vſe) certaine Nauticall Queſtions, together
with the finding of the variation of the
Compaſſe. All performed Arith-
metically, without Mappe,
Sphære, Globe, or Aſt-
rolabe, by the ſaid
R. H.
Printed for Io. Tappe.

Title-Page of the Pitiscus *Trigonometry*, 1614

identical with that for **3**. In the *Canon* each page is headed

A Table of
Sines || Tangents || Secants

but under each of these headings are two columns of figures; under Sines (for sines and cosines), under Tangents (for tangents and cotangents), under Secants (for secants and cosecants). Compare **17**.

10. From manuscript material collected by Dr. JAMES HENDERSON of the University of London, author of *Bibliotheca Tabularum Mathematicarum*, part 1(1926), I learn of the following French edition of **3**, which he had evidently inspected: *Canon Manuel des Sinus, touchantes et couppantes. Supputé par B. Pitiscus, & corrigé en ceste édition en laquelle sont aditionnées toutes les choses principales & necessaires à la Trigonometrie, extraites des traictez de la doctrine des triangles, tant rectilignes, que spheriques, faits par D. Henrion*. Paris, Abraham Pacard, 1619. According to the *Catalogue* of the Bibliothèque Nationale[14] it possesses two copies of **11**, a 1623 edition of no. **10**, published by M. Mondière in Paris, 288 p. Of course the original may have been **5**.

12. The *Catalogue* of the Bibliothèque Nationale[14] lists also another work of 1623, in Latin, which seems to be yet another edition of no. **3** (or **5**): *Sinuum, Tangentium et secantium Canon Manualis, Supputatus a B. Pitisco et emendatus in hac editione . . . Desiderii Henrioni*. Paris, apud M. Mondière, 1623, 323 p.

13. We now turn back chronologically to 1607, when the second table of Pitiscus was published. RICHE DE PRONY,[3a] DELAMBRE[8] and DEMORGAN[6] seem to be the only writers who give definite accurate details regarding this publication, which, with the last publication of Pitiscus referred to above, are more intimately linked with RHETICUS, to whose work we must now briefly refer. In 1551 RHETICUS, that is, GEORG JOACHIM of Rhetia, published in Leipzig his *Canon Doctrinae Triangulorum*[4] which gave 7D tables (14 p.) of the six trigonometric functions, at interval 10′, with differences; he here initiated our semi-quadrantal arrangement. The interval was probably chosen because of the reported statement of COPERNICUS, with whom Rheticus was intimately associated, that such tables at interval 10′ would be of value in astronomy. This is the first table in which sines, tangents and secants are joined together. VIÈTE in 1579 gave a 7D canon for every minute.

14. After 1551, for at least a dozen years before his death in 1576 Rheticus and a corps of computers carried on colossal computations in preparing the manuscript for his monumental *Opus Palatinum de Triangulis*, published 20 years after his death, at Neustadt in the Palatinate, 1596; it is so called in honor of the Elector of the Palatinate, Frederick IV, who bore the expense of publication. There are about 1440 folio pages in the whole work, tables occupying one half. There is the remarkable, complete trigonometric canon (with errata, 554 p.) to 10D, at interval 10″; to 15D with the first and last degree at interval 1″; and to 15D in tables of tangents and secants, at interval 1′. Following this great table was a 7D table of the trigonometric canon, much less accurate, and at interval 10″, evidently a preparatory ms. of Rheticus; its publication in this volume by the editor VALENTINE OTHO (1550?–1605) a pupil whom Rheticus had engaged to assist him in the year before he died, was difficult to understand. Otho became professor of mathematics at the Univ. of Wittenberg. The great canon

was all but complete at the time of Rheticus' death. Otho added about two-thirds (460 p.) of the text to one third prepared by Rheticus, and the publication of the work was a great advance in the development of trigonometry.

Shortly after the *Opus Palatinum* was published it was found "that the tangents and secants towards the end of the quadrant became more and more erroneous and at the extreme end were very erroneous indeed." Pitiscus was engaged to correct the tables. Rheticus seems to have realized that a sine or cosine table to more than 10D would be necessary for such correction. Finally after the death of Otho, Pitiscus found such a Rheticus ms. of sines and cosines to 15D, Δ^3, at interval 10″; and also a supplementary 15D, Δ^2, sine table for the first and last degrees of the quadrant, at interval 1″. Pitiscus then made two publications.

15. In the first he corrected all that part of the great table in which the tangents and secants are sensibly erroneous, being the first 86 pages. These he reprinted, and joined to the remaining pages of the great table, making 540 pages since the errata pages disappeared. He then cut away some of the text material and added a short description or *commonefactio*, as he calls it. The whole was issued with a special title page[10] *Bartholomaei Pitisci Grünbergensis Silesii Brevis Et Perspicua commonefactio De Fabrica Et Vsv Magni Canonis doctrinae Triangulorum Georgii Ioachimi Rhetici. Neostadii, Typis Nicolai Schrammii.* MDCVII. DeMorgan writes[6] that the 86 pages of reprint are easily distinguishable by the inferiority of paper and type. Riche de Prony states[3a] that he knew of only two existing copies of the 86 pages by Pitiscus, one copy which he had himself acquired, and the other listed in *Catalogue des Livres de la Bibliothèque du Conseil d'État.* Paris, v. 1, 1803, nos. 2781–2782. But this library was destroyed by the Commune in 1871.

16. In the second publication, to the Rheticus mss. which we have described above, Pitiscus added the sines of every 10^{th}, 30^{th}, 50^{th} second, in the first 35 minutes, to 22D and published the whole at Frankfurt in 1613 in a folio volume with a long descriptive title beginning *Thesaurus Mathematicus*. Pitiscus died very shortly after its publication. The manuscript of the great table in the *Thesaurus* must have been extraordinarily accurate. Only three results of examination of the printed table appear to have been published. By comparing the 14th and 15th decimal places of 400 entries of the *Thesaurus* with the 14th–20th decimal places in the 25D *Table du Cadastre*, at interval $0^g.01$ (see *MTAC*, v. 1, p. 34), Riche de Prony gives a table to show[3a] that the Rheticus 14D is always exact; that 15D is often in error by 1 unit, sometimes 2 units, and rarely 3 units—but never more. He pointed out also that in the supplementary Rheticus table, the first significant figure in the value for sin 1″ is misprinted 2 (instead of 4). A. Gernerth tested the first 8 of the 15 decimal places of the 32400 entrie- of the sine-cosine table ("Bemerkungen über ältere und neuere mathes matischen Tafeln," *Z.f.d. österr. Gymn.*, Vienna, Heft 6, 1863, p. 426–428) and found 122 errors, two of them in first differences. Of the 120 remaining errors, two of them were digit transpositions, and 118 were single digit errors. In every case the first differences given corresponded to the corrected result. This suggests, therefore, that there was not a single error in the original ms. in the first 8 places of 32400 entries. Gernerth did not list 22 of the above-mentioned 120 errors, because they had been already listed by G.

Vega, in his *Logarithmische, Trigonometrische, und andere zum Gebrauche der Mathematik eingerichtete Tafeln und Formeln.* Vienna, 1783, p. VI.

Copies of the Pitiscus edition of the *Thesaurus* which belonged in succession to Lalande, Delambre, and Babbage, and also of the Pitiscus *Opus Palatinum*, belonging to Babbage, are in the Crawford Library.[12] According to Dr. Hohenemser's *Katalog der mathematischen Abteilung* (1909) of the City of Frankfurt Library, copies of **14**, **15** are to be found there; but the Librarian informed me that these were destroyed during World War II. Nos. **1**, **5**, **6**, **14**, are in the Bibliothèque Nationale; **1**, **2**, **4–7**, **13**, **14**, are in the British Museum; nos. **2** (film), **3** (film), **4–7**, **14** (film), **15** (film), **16**, **17** (film) are at Brown University; **4**, **5** are in the Lincoln *Cathedral;* **2**, **6**, **7**, are at Columbia University; **2** is in Edinburgh Univ.; **14** is in Library of Congress; **2**, **3**, are at the University of Michigan; **16**, is in the Greenwich Observatory Lib., and **6**, **7**, are in the New York Public Lib. and Yale University Lib. This record of Library copies of Pitiscus' tables here listed makes no pretence at completeness. See further below[18] where the list is as complete as it could be made.

17. After galley proof of this article had arrived I received from the University of Cambridge Library information concerning their copy of a Pitiscus volume, which appears to be excessively rare, and unlisted in any of the ordinary bibliographical or historical sources. This volume is entitled: *Sinuum, tangentium et secantium Canon Manualis Accomodatus ad trigonometriam. Bartholomæi. Pitisci Grünbergensis Silesij. Heidelbergæ.* Typis Iohan Lancelloti, Acad. Typo. Impensis Ionæ Rosæ. MDCXIII. Signatures: $A\text{-}H^{12}$, I^{4}. [200 p.] It contains the same tables as **3**, but differently arranged, each page for sin, tan, sec, being opposite a page for cos, cot, cse, although still headed sin, tan, sec. In addition there are a 2-page Explicatio numerorum huius canonis, and an 8-page De erratis huius canonis. Hence **9a**, **9b** may have been English editions of **3**, taking account of the corrected **17**. Just as the page-proof arrived I discovered that the University of Illinois Library also has a copy of **17**, which it acquired when in 1948 it purchased the collection of fables belonging to the late Egon, Ritter von Oppolzer (1869–1907). Nos. **2**, **3**, **6**, **12**, **16** are also in this collection.

Cf. III (No. 27, July, '49) p498

R. C. A.

[1] Gass, *Allg. Deutsche Biographie*. Leipzig, v. 26, 1888.

[2] A. G. Kästner, *Geschichte der Mathematik*. Göttingen, v. 1, 1796, p. 564–565, 581–590, 612–626; v. 2, 1797, p. 743–746.

[3] Delambre, *Histoire de l'Astronomie Moderne*, v. 2. Paris, 1821, p. 26–35.

[3a] G. C. F. M. Riche de Prony, "Eclaircissemens sur un point de l'histoire des tables trigonométriques," *Mémoires de l'institut Nat. d. Sci. et Arts, Sci. Math. et Phys.*, Paris, v. 5, 1804, p. 67–93.

[4] DeMorgan, "On the almost total disappearance of the earliest trigonometrical canon," RAS, *Mo. Not.*, v. 6, 1845, p. 221–228; reprinted with an addition in *Phil. Mag.*, s. 3, v. 26, 1845, p. 517–526. Dealing chiefly with the Rheticus table of 1551, no. 13.

[5] A. DeMorgan, "Table," in *The English Cyclopaedia, Arts and Science* Sect., London, v. 7, 1861, cols., 987–990.

[6] J. W. L. Glaisher, BAASMTC, *BAAS Report 1873*, 1873, p. 1–175 "Rheticus" and "Pitiscus," p. 43–45, 158.

[7] N. L. W. A. Gravelaar, "Pitiscus' Trigonometria," *Nieuw Archief voor Wiskunde*, s. 2,

v. 3, p. 253–278, 1898. This valuable article contains complete titles, signatures, subheadings and titles for the four Pitiscus Trigonometries, nos. **1**, **2**, **4**, **6**.

[8] M. Cantor, *Vorlesungen über Geschichte der Mathematik*. Leipzig, v. 2, second ed., 1900, Pitiscus, p. 603–604, 619, 642, 646–647, etc.

[9] A. von Braunmühl, *Vorlesungen über Geschichte der Trigonometrie, Erster Teil*. Leipzig, 1900, Rheticus and Pitiscus, p. 144–148, 212–226; Teil 2, 1903, various references to Pitiscus.

[10] *Napier Tercentenary Memorial Volume*, ed. by C. G. Knott. Publ. for the R. Soc. Edinburgh, 1915; "The great tables preceding the discovery of logarithms," p. 213–218, by R. A. Sampson.

[11] F. Cajori, *A History of Mathematical Notations*, v. 1, Chicago, 1928, "Did Pitiscus use the decimal point?," p. 317–323.

[12] *Catalogue of the Crawford Library of the Royal Observatory Edinburgh*. Edinburgh, 1890. Pitiscus and Rheticus entries are here of interest to us.

[13] Our words tangent and secant are also due to a sixteenth century writer, Thomas Fincke (1561–1656), a Dane, in his *Geometriae Rotundi Libri XIIII ad Fridericum secundum*. Basle, 1583; second ed. 1591.

[14] Bibliothèque Nationale, *Catalogue Général*, Paris, v. 138, 1936, entries under Pitiscus.

[15] A. W. Pollard & G. R. Redgrave, *A Short-Title Catalogue of Books Printed . . . 1475–1640. . . .* London, 1926. *STC*. Two errors in Pitiscus listings.

[16] W. W. Bishop, *A Checklist of American Copies of "Short-Title Catalogue" Books*. Ann Arbor, Mich., 1944. *STCA*.

[17] C. K. Edmonds, *Huntington Library Supplement to the Short Title Catalogue, Huntington Lib. Bull.*, no. 4, 1933. *STCH*.

[18] English Pitiscus Entries[15–17] Revised

[19966a]. *A Canon of Triangles*, [1614], entered in the Stationers' Company Register 17 Jan. 1614. No place of publication, no printer's name, no date. No entry in *STC;* here identified for the first time. Signatures A-L^4, M^2.
Library Copies: Brown Univ., Mr. Harrison D. Horblit of New York, Huntington Lib.

19966. [Anr. ed.] 1630. 4to. T. Purfoot for J. Tapp, 1630.
Library Copies: Boston Public Lib., British Museum (omitted in *STC*), Univ. Cambridge, Crawford Lib., Huntington Lib., Univ. Michigan, Yale Univ.

19967. *Trigonometry*. Tr. Ra: Handson, 1614.
Library Copies: Mr. Harrison D. Horblit, Huntington Lib., Lincoln Cathedral. Not in the British Museum as stated in *STC*.

19968. [Anr. ed.], 1630.
Library Copies: Boston Public Lib., British Museum (omitted in *STC*), Huntington Lib., Yale Univ.

19968a. [Anr. ed.], [1631]. Entered in the Stationers' Companys' Register 1 Aug. 1631.
Library Copies: Univ. Cambridge, Crawford Lib., Univ. Illinois, Univ. Michigan. Not in the Huntington Lib. (*STCH*), as stated in *STC*.

97. Fritz Emde.—Few names are more familiar to the mathematician than those of Jahnke & Emde, to various editions of whose *Tables of Higher Functions*, since the first in 1909, we have frequently made reference. Indeed the last German edition, surveyed elsewhere in this issue, appeared shortly after Professor Emde's seventy-fifth birthday, July 13, 1948. Since Jahnke died in 1921, to Professor Emde's genius belong the greatly enlarged and improved editions of the *Tables* after the first. In 1912 Professor Emde was appointed Professor of Electrotechnics and Director of the Electrotechnic Institute in the Technische Hochschule, Stuttgart. And now as professor emeritus (nominally since 1938 but actually since 1943) he still keeps in

touch with scientific activities, although he has almost entirely lost his eyesight. He received honorary degrees of Doctor of Engineering from the Technische Hochschule, Breslau, in 1913, and from the Eidgenössische Technische Hochschule, Zurich, in 1929. We are happy to present a portrait of Professor Emde taken about the time (1938) of publication of the third edition of his *Tables of Higher Functions*. Last month a new edition of his *Tafeln elementarer Funktionen* (see MTAC, v. 1, p. 384–385) was published in Germany.

98. Mersenne Numbers.—In Nat. Acad. Sci., *Proc.*, v. 34, Mar. 1948, p. 102–103, Professor H. S. Uhler gives details of his proof (completed 27 Nov. 1947) that M_{193} is composite. Thus he brought to a conclusion work begun in 1944 (*MTAC*, v. 1, p. 333) when the characters of just six of the M_p, $p = 157, 167, 193, 199, 227, 229$, were unknown. He has now shown that all of these are composite. See also *MTAC*, v. 1, p. 404; v. 2, p. 94, 341. Professor Uhler's final summary of some of the facts concerning the 55 Mersenne numbers is as follows:

p	Character of M_p
2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127	Prime
11, 23, 29, 37, 41, 43, 47, 53, 59, 67, 71, 73, 79, 113	Composite and fully factored
151, 163, 173, 179, 181, 223, 233, 239, 251	Two or more prime factors found
83, 97, 131, 167, 191, 197, 211, 229	Only one prime factor known
101, 103, 109, 137, 139, 149, 157, 193, 199, 227, 241, 257	Composite but no factor known

R. C. A.

QUERIES

29. Pitiscus Tables.—Where may one consult other copies of English or French Pitiscus tables, of the 1613 Pitiscus tables, and of the 1607 Rheticus-Pitiscus table, than those listed in **N96**?

R. C. A.

QUERIES—REPLIES

38. Log Log Tables (Q**4**, v. 1, p. 131; QR**9**, p. 336, **12**, p. 373; **30**, v. 2, p. 374).—The following tiny publication of a "professeur à la Faculté des Sciences de Paris" and an "ingénieur civil des mines" contains a 4D table of log log N, for $N = 1.003(.001)1.2(.01)2(.1)10(1)100(10)1000(100)$-$10000(1000)39000$. . . : Jean Villey & Jean Dienesch, *Table des Logarithmes de Logarithmes. Jointe à une table de logarithmes ordinaire, permet d'effectuer très rapidement les calculs thermodynamiques* pv^k; *les calculs d'intérêts composés* $(1.03)^n$; *et tous calculs d'exponentielles* n^h. Paris, Gauthier-Villars, 1942. 8-page folded card. 8 × 13.7 cm. 7.50 francs.

CORRIGENDA

V. 1, p. 64, *for lines-(11–13)*, *read* (a′ − b′) Its semiquadrantal arrangement with sines and cosines on the same page; p. 160, l. −8, *for* 8.772, *read* 8.771; p. 298, l. −4, *for* 151, *read* 156; p. 386, l. 33, *for* 229(6), *read* 229(8), and *for* 239(10), *read* 239(17).

V. 2, p. 36, in equations (1) and (2), *for* $e^{2\pi ir}$, *read* $e^{-2\pi ir}$; p. 380, l. 27, *for* 296,357, *read* 296, 309–312, 357; p. 381, l. 12, *for* 56, 65, *read* 56, 65, 87.

V. 3, p. 186, l. 7, *for* **537**, *read* **535**; p. 225, l. 9, *for* a new one substituted., *read* a new one substituted, and an important new anonymous 16-page Appendix, apparently written by William Oughtred.

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